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# Impact of Brownian Motion and Thermophoresis on Casson Nanofluid flow in an Annular Microchannel

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***Abstract***

*The current examination delves into the Casson fluid flow in an annular microchannel by employing the Buongiorno model. The viscous dissipation effect and magnetic field effect is analyzed during the study in presence of two prominent slip mechanisms namely Brownian motion and thermophoresis in an annular microchannel incorporated with porous media. The governing equations are modeled based on these effects and are non-dimensionalized with the aid of non-dimensional variables. The non-dimensional equations formed are numerically computed using Runge Kutta Fehlberg 4-5$^{th}$ order method and graphically illustrated. Results of this study show that velocity profiles enhance for increasing values of Casson parameter and Darcy number. Nusselt number is increasing function of Brownian motion parameter and fluid wall interaction parameter.*

***Keywords:*** *Casson fluid, Buongiorno model, Darcy Forchheimer model, convective boundary, annular microchannel*

## 1. Introduction:

Variety of high density, high power and high-speed microelectronic devices require high rate of heat transfer. To operate such electronic devices at optimum temperature it is necessary to develop effective heat removal methods. One such method is to use microchannel heat sinks. A microchannel heat sink is a structure with many microscale channels of large aspect ratio built on the back of the microchip. The concept of the microchannel heat sinks was introduced by Tuckerman and Pease [1]. Abu-Nada et al. [2] studied natural convection of the fluid using different types of water-based nanofluids. They observed that addition of nanoparticles into base fluid enhances the heat transfer. Number of studies on Brownian motion and thermoporesis is carried out.

Buongiorno [3] proposed a model in which he explains two prominent slip mechanisms between nanoparticles and base fluids. The steady boundary layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream is investigated by Bachok et al [4]. In their case the plate moves in the same or opposite direction to the free stream. Recently, Pakravan and Yaghoubi [5] worked on thermophoresis, Brownian motion effect on natural convective heat transfer of nanofluids simultaneously. They showed the effect of Dufour strongly decreases with temperature. Gireesha et al [6] carried out the numerical simulation on MHD flow over a stretching surface. Sheikholeslami et al [7] carried out a study on the same model for the unsteady nanofluid flow.

They have shown that skin-friction co-efficient has direct proportionality with squeeze number. Effect of viscous dissipation on MHD flow of Casson liquid over a vertical stretching plate has been studied by Gireesha et al [8]. They found that for higher values of magnetic parameter, momentum boundary layer thickness diminishes. Almeida et al [9] looked into micropolar fluid flow in a microchannel using Buongiorno model.

Akbar et al [10] studied the impact of magnetic field on Casson fluid and obtained that rise in values of magnetic parameter leads to increase of velocity profile. MHD flow Casson fluid over a moving cylinder of linear velocity has been investigated by Tamoor et al.[11]. Convection flow of Casson fluid through the microchannel has been studied by Khan et al [12]. They adopted Caputo–Fabrizio fractional derivative method to find the solution. Kumar et al [13] deliberated the mixed convection flow of Casson fluid over a vertical plate considering nonlinear radiation. It is shown that, temperature increases as the Dufour effect and Soret parameter increases.

Reddy et al [14] investigated the thermal analysis of MHD electro-osmotic peristaltic pumping of Casson fluid through a rotating asymmetric microchannel. They found that Casson fluid velocity, temperature interaction of annular flow in microchannel boiling. They found that expansion of the bubble will make the notch at the end of the annulus. Natural convection flow of Casson fluid through an annular microchannel formed by two cylinders in the presence of magnetic field has been studied by Gireesha and Sindu [21]. It concluded that higher values of Casson parameter increases the skin friction coefficient. Further it is obtained that rate of heat transfer diminishes as fluid wall interaction parameter increases.

Nonlinear convection study on flow of a dissipative Casson nanofluid through porous and heat transferred rate are enhanced with a boost in electro-osmotic force. Jha and Malgwi [15] carried out computational analysis for Casson and Maxwell flows. Venkatesh et al [16] studied the Darcy Forchheimer flow of Casson fluid in microchannel by implementing Buongiorno model. Further, Felicita et al [17] analyzed mixed convective flow of same fluid in presence of couple stress effects of fluid particles.

Jha and Aina [18] investigated the influence of induced magnetic field on fully developed convection flow in an annular microchannel. It is obtained that for induced magnetic field, skin friction at the annular microchannel surface is higher. Taiwo and Dauda [19] gave away the result for impact of heat generation/absorption in an annulus incorporated with porous media. Their study reveals that the flow in the system can be controlled by selecting appropriate value of flow parameter as well as the outer cylinder. This mechanism can be found in annular fin where heat loss is found to be maximum when fin is isothermal. Also, Liu and Wang [20] came up with interesting results on fluid flow in annulus of channel. They studied bubble.

The present analysis reveals the impact of Brownian motion and thermophoresis on Casson nanofluid flow in an annular microchannel. Impact of magnetic field on the flow of Casson nanofluid with simultaneous transport of heat and mass is analyzed. The flow is carried out in presence of porous media supporting the flow of the fluid. The velocity slip and temperature jump boundary conditions are employed. The non-linear equations so attained are solved with the aid Runge Kutta Fehlberg $4-5^{\text{th}}$ order method in association with shooting technique. The medium of an inclined annular microchannel was studied by Idowu et al [22]. They showed that nonlinear convection parameter decreases the volume fraction whereas it increases the energy distribution. Escandón et al [23] investigated the electro osmotic multi-layered flow of viscoelastic fluids in annulus of the microchannel. They have made use of Poisson-Boltzmann equation and hydrodynamic boundaries at liquid-liquid and solid liquid interfaces. Their study has prominent implications for control of fluid transport in microfluidic devices. Li and Hrnjak [24] modeled a mechanistic model for flow in an microchannel tube to envisage the heat transfer co-efficient and gradient of pressure.

The radius of inner cylinder is $r_1$ and that of outer cylinder is $r_2$. The outer surface of the inner cylinder is heated to a temperature $T_1$ in such a way that the surrounding is having temperature $T_0 < T_1$.

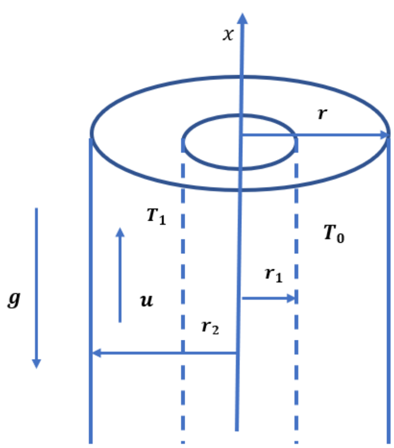


Figure 1: Geometry of the flow problem

Here the inner surface of the outer cylinder is maintained at temperature $T_0$. Natural convection occurs due to this temperature difference. Due to this flow is fully developed. Cylinders are of infinite length; the transport phenomenon depends only on $r$. A uniform magnetic field of strength $B_0$ is applied normal to direction of flow. Migration of nanoparticle exists in nanofluid because of the variation in velocity and temperature profile for different parameter is examined and is displayed through graphs. Skin-friction co-efficient, Nusselt number, Sherwood number illustrations of graph portray the prominent outcomes of the examination.

## 2. Mathematical Formulation:

Natural convection flow of an incompressible Casson fluid in a vertical annular microchannel formed by two concentric cylinders with porous medium is considered. Let $X$ axis is parallel to gravitational field $g$ in opposite direction while $r$ axis is in the radial direction.

$$\left(1+\frac{1}{\beta}\right)\frac{\nu_f}{r}\frac{d}{dr}\left[r\frac{du}{dr}\right]-\sigma_f\frac{B_0^2 u}{\rho}+\frac{\nu_f}{k_p}\left(1+\frac{1}{\beta}\right)u=0, \qquad (1)$$

$$\frac{D_B}{r}\frac{d}{dr}\left[r\frac{dC}{dr}\right]+\frac{D_T}{T_0}\frac{d^2T}{dr^2}=0 \qquad (2)$$

$$\frac{\alpha}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right]+\tau\left[D_B\frac{dT}{dr}\frac{dC}{dr}+\frac{D_T}{T_0}\left(\frac{dT}{dr}\right)^2\right]+\left(1+\frac{1}{\beta}\right)\frac{\nu_f}{\rho C_p}\left(\frac{du}{dr}\right)^2=0 \qquad (3)$$

The boundary conditions for both velocity and temperature field are[15] [21]:

$$u==\frac{2-\sigma_v}{\sigma_v}\lambda\frac{du}{dr}\bigg|_{r=r_1} \quad \text{at } r=r_1$$

$$T=T_1+\frac{2-\sigma_t}{\sigma_t}\frac{2y}{2y+1}\frac{\lambda}{Pr}\frac{dT}{dr}\bigg|_{r=r_1} \quad \text{at } r=r_1 \qquad (4)$$

$$u=-\frac{2-\sigma_v}{\sigma_v}\lambda\frac{du}{dr}\bigg|_{r=r_2} \quad \text{at } r=r_2$$

$$T=T_0+\frac{2-\sigma_t}{\sigma_t}\frac{2y}{y+1}\frac{\lambda}{Pr}\frac{dT}{dr}\bigg|_{r=r_2} \quad \text{at } r=r_2 \qquad (5)$$

where, $\beta$ is Casson paratmeter, $D_T$ is thermophoresis diffusion co-efficient, $D_B$ is a Brownian diffusion co-efficient, $\alpha$ is thermophoresis and Brownian motion. The velocity slip and temperature jump boundary conditions are employed.

The governing equations comprising momentum and energy for Casson nano fluid as follows [15] [21]:
On introducing following dimensionless quantities

$$R=\frac{r-r_1}{W}, W=r_2-r_1, U=\frac{u}{u_c}, \theta=\frac{T-T_0}{T_1-T_0}, \eta=\frac{r_1}{r_2}, u_c=\frac{\rho g^*\beta_0(T_1-T_0)}{\mu_0}W^2,\ \phi=\frac{c-c_\infty}{c_1-c_\infty} \quad (6)$$

From equation (6), equations (1)-(3) becomes

$$\left(1+\frac{1}{\beta}\right)\left[\frac{d^2U}{dR^2}+\frac{(1-\eta)}{(R(1-\eta)+\eta)}\frac{dU}{dR}\right]-M^2U+\frac{1}{Da}\left(1+\frac{1}{\beta}\right)U=0 \quad (7)$$

$$\frac{d^2\theta}{dR^2}+\frac{(1-\eta)}{(R(1-\eta)+\eta)}\frac{d\theta}{dR}+\left(1+\frac{1}{\beta}\right)\mathrm{EcPr}\left(\frac{du}{dR}\right)^2+Nb\frac{d\theta}{dR}\frac{d\phi}{dR}+Nt\left(\frac{d\theta}{dR}\right)^2=0 \quad (8)$$

$$\frac{d^2\phi}{dR^2}+\frac{(1-\eta)}{(R(1-\eta)+\eta)}\frac{d\phi}{dR}+\frac{N_t}{N_b}\frac{d^2\theta}{dR^2}=0 \quad (9)$$

where, $\beta_t=\frac{2-\sigma_t}{\rho_t}, \beta_\nu=\frac{2-\sigma\nu}{\sigma\nu}$ are dimensionless variables, $M^2=\frac{\sigma B_0^2W^2}{\rho_0\nu}$ is Hartmann number, $Da=\frac{K}{w^2}$ is Darcy number, $Kn=\frac{\lambda}{W}$ is Knudsen number, $Ln=\frac{\beta_t}{\beta_\nu}$ is fluid wall interaction parameter, $E_c=\frac{g^2\rho^2(T_1-T_0)}{\nu C_p}$ is Eckert number, $Pr=\frac{\nu}{\alpha}$ is Prandtl number, $Nb=\frac{\tau D_B(C_1-C_\infty)}{\alpha^*}$ is the Brownian motion parameter and $Nt=\frac{\tau D_T(T_h-T_a)}{T_a\alpha^*}$ is the thermophoresis parameter. The corresponding boundary conditions are as follows:

thermal diffusivity, $\nu$ is fluid kinematic viscosity, $\sigma_t$ is tangential momentum accommodation coefficients, $C_p$ specific heat at constant pressure, $\sigma_\nu$ is thermal momentum accommodation coefficients.

$$\theta(0)=1+\beta_\nu\,\mathrm{Knln}\left.\frac{d\theta}{dR}\right|_{R=1} \quad \text{at } R=0 \quad (10)$$

$$U(1)=-\left(1+\frac{1}{\beta}\right)\beta_\nu\,\mathrm{Knln}\left.\frac{d\theta}{dR}\right|_{R=1} \quad R=1$$

$$\theta(1)=-\beta_\nu\,\mathrm{Knln}\left.\frac{d\theta}{dR}\right|_{R=1} \quad \text{at } R=1 \quad (11)$$

## 2.1 Quantities of Physical Significance

The parameter of physical interest are Nusselt number, skin friction and Sherwood number which are given by [21],

$$Nu=-\left.\frac{d\theta}{dR}\right|_{R=1} \quad (12)$$

$$Cf=\left.\frac{dU}{dR}\right|_{R=1} \quad (13)$$

$$Sh=-\left.\frac{d\phi}{dR}\right|_{R=1} \quad (14)$$

here $Nu, Cf$ and $Sh$ are the Nusselt number, skin friction and Sherwood number at the cylinder surface.

## 3. Results and Discussion

In this section we analyze the nature of velocity profile, temperature profile, skin friction, Sherwood number and Nusselt number for different flow parameters such as rarefaction parameter $\beta_\nu$Kn, fluid wall interaction parameter $ln$, magnetic

$$U(0) = \left(1 + \frac{1}{\beta}\right)\beta_v Kn \left.\frac{dU}{dR}\right|_{R=0} \quad \text{at } R = 0$$

parameter $M$, Darcy number $Da$, Casson parameter β, Brownian motion $Nb$ and thermophoresis parameter $Nt$ with the help of graphical representation. Figure 2 shows the impact of magnetic parameter $M$ on the velocity profile. As the values of magnetic parameter $M$ increases fluid velocity decelerates. This is due to the fact that the strong magnetic field develops Lorentz force (a resistive kind of force) which slows down the flow velocity. Influence of Casson parameter $\beta$ on velocity profile is depicted in figure 3. Higher value of Casson parameter leads to the enhancement of velocity profile as $\beta$ tends to infinity the fluid neutralizes itself from non-Newtonian to Newtonian category. Figure 4 exhibits the impact of Darcy number $Da$ on-velocity profile. It is shown that velocity profile increases for increase in Darcy number. This is because, permeability of porous medium enhances, leading to the cut down in friction with this medium. Figure 5 represents the impact of rarefaction parameter $\beta_v Kn$ on temperature profile. Temperature profile decreases as rarefaction parameter $\beta_v Kn$ increases. This happens due to less interaction of fluid molecules and hot wall of the cylinder. Figure 6 portrays the influence of fluid wall interaction parameter ln on temperature profile. It observed that as fluid wall interaction parameter increases temperature decreases.

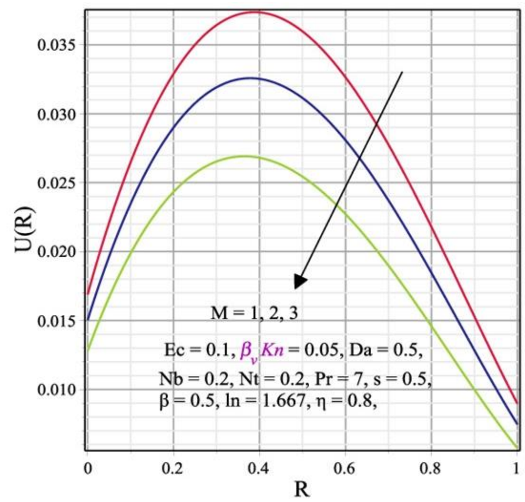


Figure 2: Influence of magnetic parameter on velocity profile.

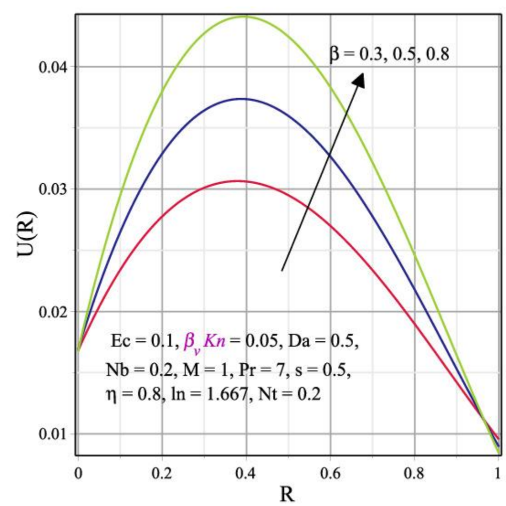


Figure 3: Influence of Casson parameter on velocity profile.

Figure 7 and 8 display the influence of Brownian motion and thermophoresis ($Nb$) on temperature. As there is increase

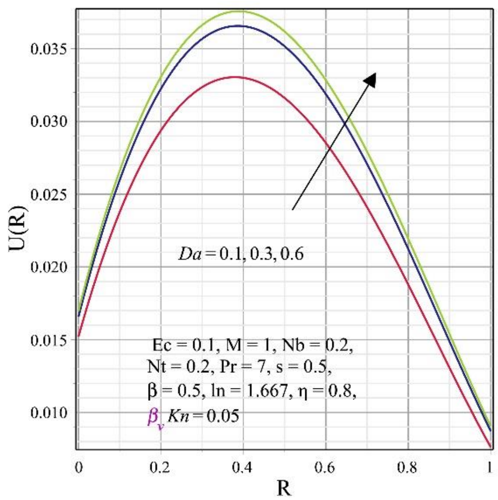


Figure 4: Influence of Darcy number on velocity profile.

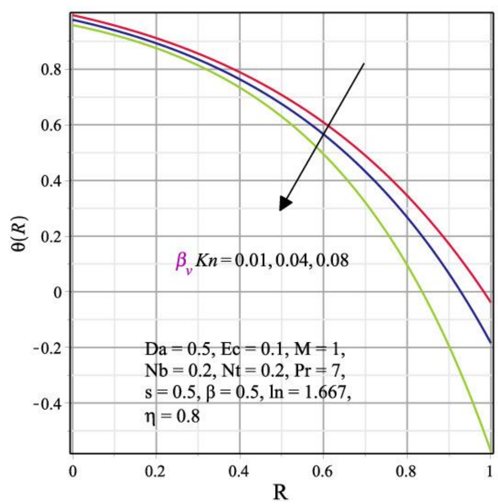


Figure 5: Influence of rarefaction parameter $\beta_v\ Kn$ on temperature profile.

in Brownian motion and thermophoresis, we observe that temperature increases at inner cylinder and decreases at outer cylinder, this is due to continuous collision of nanoparticles generates thermal energy, coupled with particles movement from a higher temperature region to lower region which increases the fluid temperature. At the outer wall the $Nb$ parameter with its increase decreases thermal profile. Concentration profiles for varying $Nb$ and $Nt$ are displayed in figure 9 and 10.

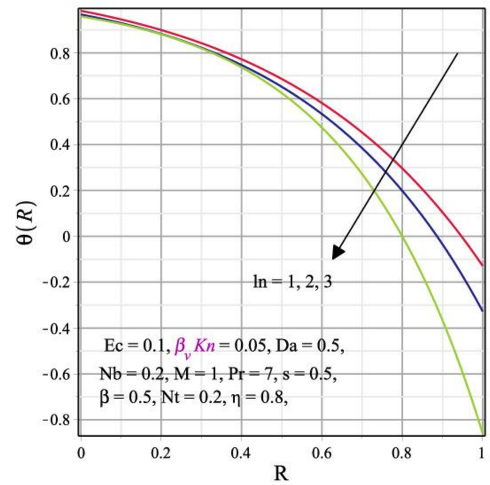


Figure 6: Influence of fluid wall Interaction parameter $ln$ on temperature profile.

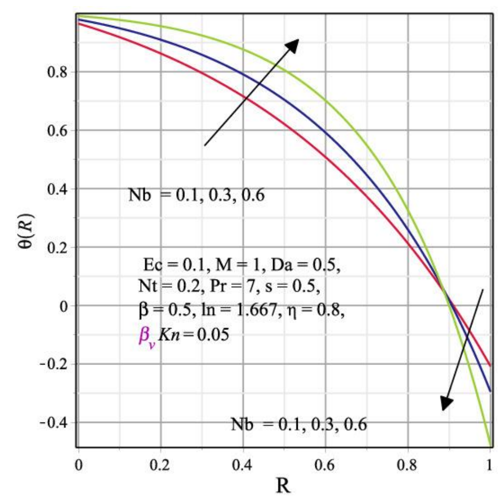


Figure 7: Influence of Brownian motion parameter $Nb$ on temperature profile

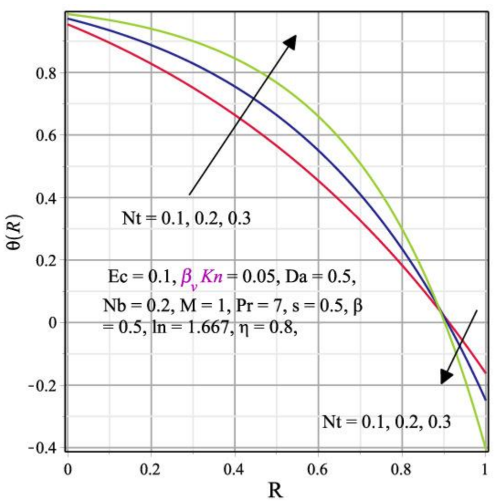


Figure 8: Influence of thermophoresis parameter $Nt$ on temperature profile.

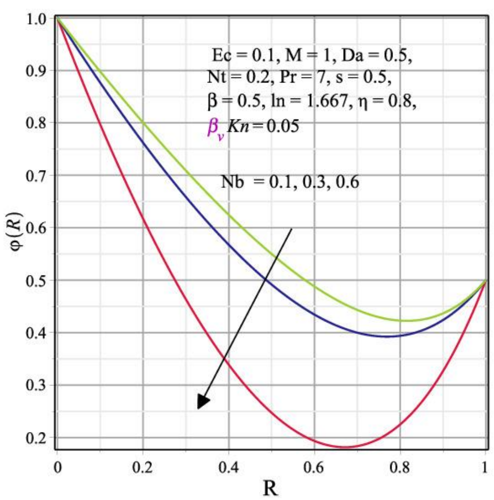


Figure 9: Influence of Brownian motion parameter $Nb$ on concentration profile.

For an increasing values of $Nb$ and $Nt$ we observe the decreasing concentration profile respectively. Multiple values of $Nb$ entirely promotes the energy, momentum and nanoparticle volume fraction profiles. Here continuous increase of $Nt$ accelerates concentration profiles. The diffusion of the nanoparticles causes movement of nanoparticles across the cylinder surfaces which results in increase of nanoparticle concentration with an increase in $Nt$. Influence of Casson parameter $\beta$ and radii ratio parameter $\eta$ on skin friction is depicted in figure 11. Higher the value of Casson parameter lesser is the skin friction.

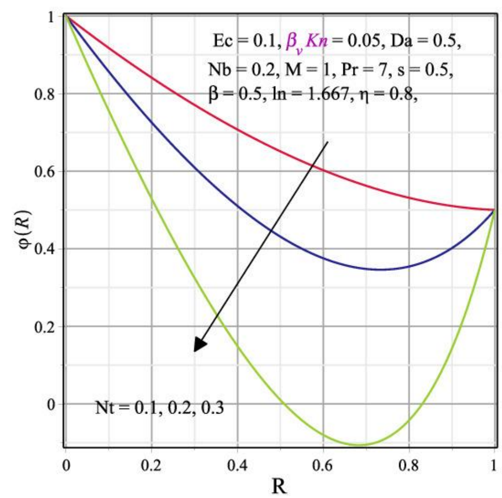


Figure 10: Influence of thermophoresis parameter $Nt$ on concentration profile.

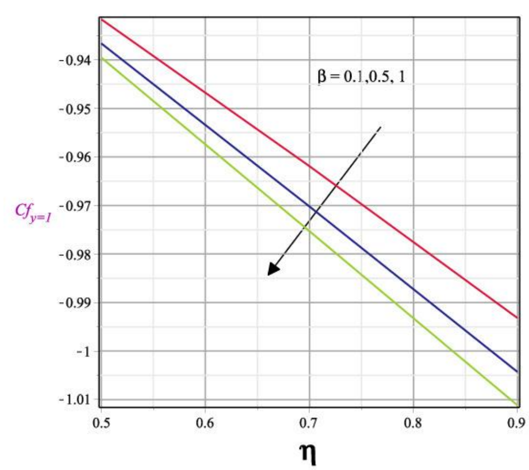


Figure 11: Influence of Casson parameter $Nb$ on skin friction profile.

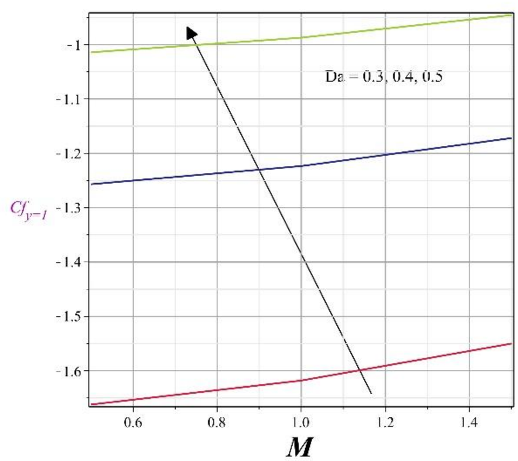


Figure 12: Influence of Casson parameter $Nb$ on skin friction profile.

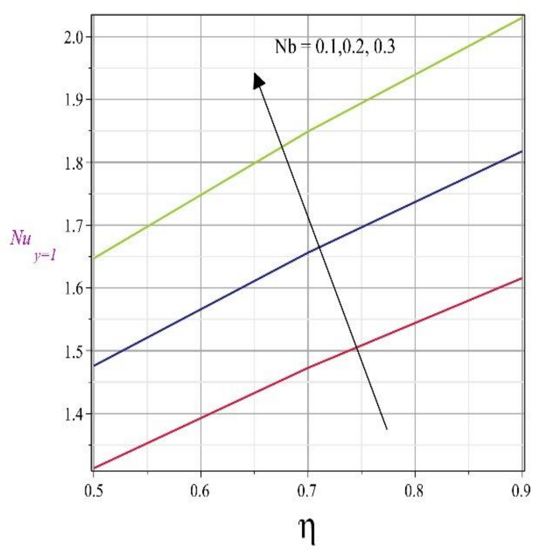


Figure 13: Influence of Brownian motion parameter $Nb$ on Nusselt number profile.

For higher value of radii ratio parameter skin friction is the least. That is enlargement of the width of annular surfaces depletes the skin friction of the fluid. Variation of skin friction coefficient for increasing values of Darcy parameter against magnetic parameter $M$ is shown in figure 12. More the Darcy number value more is the skin friction and less the magnetic field applied lesser is the skin friction co-efficient. Characteristics of Nusselt number for varying Brownian motion parameter is represented in figure 13.

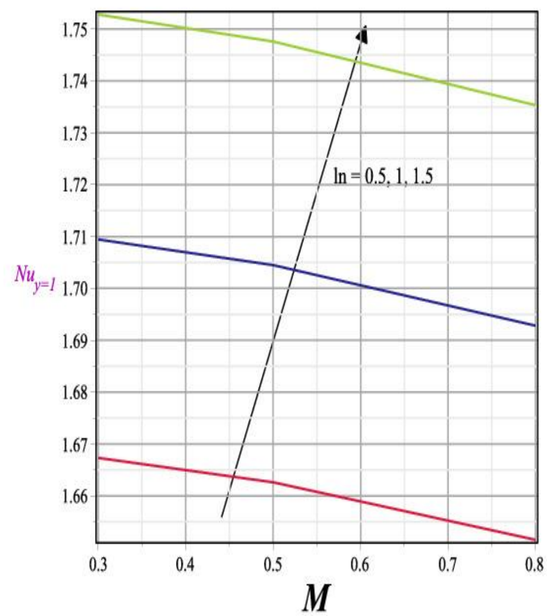


Figure 14: Influence of Brownian motion parameter $Nb$ on Nusselt number profile.

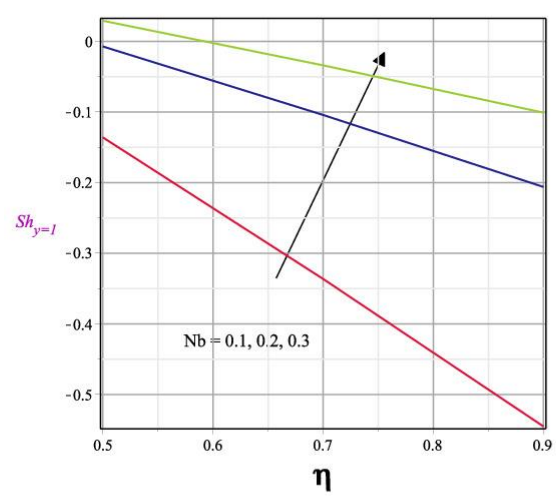


Figure 15: Influence of Brownian motion parameter $Nb$ on Sherwood number profile.

It shows that Nusselt number increases for increasing values of Brownian motion parameter $Nb$ when plotted against radii ratio parameter. And it is also observed that Nusselt number increases for increasing values of fluid wall interaction parameter $ln$ when plotted against magnetic parameter $M$, it is shown in figure 14.

It is obtained that Sherwood number shows increasing trend for increasing values of Brownian motion parameter. Also, for least $Nb$ value mass transport rate is less. Impact of fluid wall interaction parameter $ln$ on Sherwood number $Sh$ is represented in figure 16, which shows decreasing trend for rising values of $ln$.

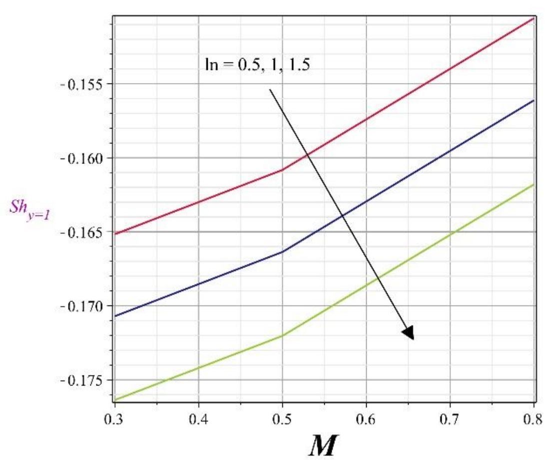


Figure 16: Influence of Brownian motion parameter $Nb$ on Sherwood number profile.

Figure 16 exhibits the impact of $ln$ on Sherwood number $Sh$ against magnetic parameter $M$.

## 4. Concluding Reviews:

The study examines the flow of Casson nanofluid in an annular microchannel when porous media is incorporated. The model employed here is Buongiorno model which exemplifies the impact of Brownian motion and thermophoresis on Casson nanofluid flow in annular microchannel. The non-linear governing equations are solved via Runge Kutta Fehlberg $4-5^{\text{th}}$ order method. Key outcomes of the present analysis are as follows:

- Higher rarefraction parameter enhances the flow velocity of the Casson fluid.
- Magnetic parameter slowdown the fluid flow through annular microchannel.
- Velocity profiles enhances for increasing values of Casson parameter and Darcy number.
- Temperature profile decreases for increasing values of rarefaction parameter and fluid wall interaction parameter.
- Brownian motion and thermophoresis parameter is decreasing function of fluid concentration.
- Casson parameter decreases for increasing values of skin friction co-efficient.
- Darcy number affects skin friction coefficient positively.
- Sherwood number increases for Brownian motion parameter.
- Nusselt number is increasing function of Brownian motion parameter and fluid wall interaction parameter.

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