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Error in Image Processing using Fourier Transform and Fast Fourier Transform

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Abstract

This manuscript aims to find out the error in the operations of Fourier Transform (FT) and Fast Fourier Transform (FFT) in digital image processing. We apply FT to transform the time-domain signal to the frequency domain signal. FT is a basic technique in the field of Mathematical and Engineering works. We use Fourier analysis in digital image processing. The Fourier analysis represents a lot of things such as filters, transformation, representation, encoding, data processing, and valid more fields. The FFT development in recent times is very important for the case of image processing. FFT is very exquisite and ubiquitous work in Enfield of digital image processing. In this paper, we study FT and FFT to demonstrate how it solves relative technology problems in the field of security of images. Also, we analyzed the error when changing the fraction order inside FFT, what changes in the mean square error, and on which fraction order our mean square error is obtained the least.

Keywords: *Digital Image, Mean Square Error, Fourier Transform, Fast Fourier Transform*

1. Introduction

FT is very important for conducting Fourier spectroscopy; this alone will explain its importance. FT is also very suitable for many other tasks such as electrical signal analysis, diffraction, optical testing, optical processing, imaging, holography, and also for remote sensing. The main purpose behind FT is to change the time domain signal to the frequency domain signal [1]. The Fourier arrangement is a strategy for communicating an intermittent capacity as several sinusoids. The Fourier Transform is subsequently an expansion of this plan to non-occasional capacities also. This causes the Fourier to change tremendously compelling in conduction of warmth, wave engendering, computerized signal handling, picture preparing, separating, and so forth [2,3]. In the space of sign handling, discrete Fourier change (DFT) is quite possibly the most basic calculation. Also, the FFT is the quickest DFT calculation. In numerous data applications, a signal has a specific level of sparsity or compressibility. Furthermore, it exists in numerous spaces, for example, picture processing1, packed detecting, computational learning hypothesis, multi-scale analysis, and so on [4]. The approach of quick Fourier change (FFT) has made spectra estimation simpler and more effective. By and by, nonetheless, the intrigued reasonable signals, for example, inadequate moving component courses are typically cyclostationary and non-fixed [5].

Fourier change (FT) is named in the honor of Joseph Fourier (1768-1830), one of the most noteworthy names throughout the entire existence of science and physical science. Fourier changes have been broadly utilized in picture handling applications, for example, picture investigation, picture sifting, picture reproduction, and picture pressure. Up to this point, there was no meaning of a Fourier change appropriate to shading pictures in a comprehensive way [6]. The Fourier change of a picture is a breakdown of the picture into its recurrence or scale parts. Such channels work on the abundancy range of a picture and eliminate, constrict, or intensify the amplitudes in indicated wavebands. Any capacity that occasionally rehashes the same thing can be communicated as several sines and cosines of various frequencies each increased by an alternate coefficient is known as a Fourier arrangement. FFT-based Image handling has arrived at a bottleneck where further speed improvement from the algorithmic point of view is troublesome. Be that as it may, some continuous applications request quicker Fourier change than what is presently accessible. Would it be advisable for us to stop our excursion for questing the Faster Fourier Transformation strategy because of algorithmic restrictions? That triggers the mission for a quicker method to register the Fourier Transform-based picture handling strategy [7]. In digital processing, FT is a very important part of the progress that is being made within communication, audio, and image processing. As an alternative method of transfer, we use the discrete signal as a function of frequency. Several terms have been defined inside the FT to continuous and discrete signals: Fourier Transform, Fourier series Discrete-Time Fourier Transform (DTFT), Discrete Fourier Transform (DFT), and Discrete Fourier Series (DFS). The only transform used for signal processing is the Discrete Fourier Transform (DFT). This last requires countless duplications and augmentations, which makes it extremely hard to carry out in inserted frameworks. A streamlining strategy is, consequently, needed to work with estimations. FFT Method Created by 1965 Cooley and Tukey. Apart from these, a lot of algorithms are built based on Cooley and Tukey's approach. The prime factor of algorithms are split radix, vector radix, vector split radix, Winograd Fourier transforms, integer FFT, etc [8]. Practically, the FFT disintegrates the arrangement of information to be changed into a progression of more modest informational collections to be changed. At that point, it disintegrates those more modest sets into much more modest sets. At each phase of preparing, the consequences of the past stage are consolidated in an extraordinary manner [9]. Quick Fourier changes (FFTs) are quick calculations, i.e., of low intricacy, for the calculation of the discrete Fourier change (DFT) on a limited abelian bunch which, thusly, is an extraordinary instance of the Fourier change on a locally smaller abelian bunch. The FFTs are among the main calculations in applied and designing math and in software engineering, specifically for one and multidimensional frameworks hypothesis and sign preparing [10].

2. Fourier Transform

Encryption is a very old method of hiding data, in this process; our information becomes safe that with the help of keys from unauthorized people, who are in the form of our text and photos. Nowadays FT is used in many applications. Has increased use, because is very easy than other methods. So it is used a lot nowadays for keeping photos safe [11]. The first lecture of FT was given by Jean Baptiste Joseph Fourier, in 1807. After this FT started being used in all areas of science and engineering. FFT is introduced in normal state FT which increases the area of FT, it is used in many places such as signal processing optics and quantum mechanics. FT is the general state of the Fourier series [12].

Which is used to describe FT in a constant and a periodic function. How do we interpret FT by the const exact function x(t) [13]

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{+\infty} x(t) \,.\, e^{-jwt} dt \quad (1)$$

We define inverse Fourier Transform by using Fourier Transform

$$X(t) = F^{-1}\{x(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} x(\omega) \,.\, e^{jwt} d\omega)$$

(2)

3. Fractional Fourier Transform

FFT as a Classical Fourier Transform into a general form was introduced as an aspect of mathematics as Namias is first and foremost in many applications of optics [14]. We need a new tune to analyze a time frequency in which FFT will play a good role. Which is the optical implementation of the FFT that was given by Mendlovic and Ozaktas in 1993, Which is used a lot in the optical field. In the year 2000, FFT was used in image encryption by G.Unnikrishnan, etc happened for the first time. FFT gives us more freedom in doing image encryption and in increasing the size of the key. FFT technique, people trend suggests that I like encryption field [15].

We will use FFT on a plane image I(x) as follows: [16]

$$\mathbf{Fp\{I(x)\}(u)} = \int_{-\infty}^{+\infty} \boldsymbol{k_p(x,u)I(x)dx} \quad (3)$$

Where KP (x,u) is the kernel function and is expressed as

$$\boldsymbol{KP(x,u)} = \begin{cases} \mathbf{T\,exp\ [i\pi(x^2cot\emptyset - 2xu\,csc\emptyset + u^2cot\emptyset)]} & \boldsymbol{p \neq n\pi}; \\ \boldsymbol{\delta(x-u)}, & \boldsymbol{p = 2n\pi} \quad (2) \\ \boldsymbol{\delta(x+u)}, & \boldsymbol{p = (2n+1)\pi}. \end{cases} \quad (4)$$

Here,

$$T = \frac{\mathbf{exp}\ [-\boldsymbol{i}\left(\boldsymbol{\pi\ sgn}\frac{\emptyset}{4} - \frac{\emptyset}{2}\right]}{\sqrt{|\boldsymbol{sin}\emptyset|}} \quad (5)$$

where $\emptyset = p\pi/2$. FFT is integral transforms which is the setup of linear and optical. We get some important property from FFT. FFT two dimension is a simple extension of one dimension which we represent as

$$\boldsymbol{F^{\alpha1,\alpha1}\{I(x,y)\}(u,v)\alpha} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \boldsymbol{K_{\alpha1,\alpha2}(x,y;u,v)f(x,y)dxdy} \quad (6)$$

Where

$$\boldsymbol{K_{\alpha1,\alpha2}(x,y;u,v) = K_{\alpha1}(x,u)K_{\alpha2}(y,v)} \quad (7)$$

4. Mean Squared Error

Mean square error (MSE) is the difference between our actual image and the recovered image, which we use to measure the quality of the image [17]. The closer to zero the rating of MSE, the better is the quality of the image [18]. MSE measures the easiest and most common distortion, the smaller the measurement of our MSE, the better the result [19,20]. MSE is given by the formula

$$\boldsymbol{MSE} = \frac{\mathbf{1}}{\boldsymbol{M-N}}\sum_{\boldsymbol{i=0,j=0}}^{\boldsymbol{M-1,N-1}}[\boldsymbol{I(i,j) - K(i,j)}]^{\mathbf{2}} \quad (8)$$

Which is our M and N is the width and length of the images, out of which I(i,j) is recovered image and K(I,j) input image in which i and j are row and columns of the input image and recovered image together.

Figure1 describes the image processing through the FFT.

If the image is the same, then its MSE is zero [21]. We checked in practical by Matlab simulation as we increase the value of the fraction order the value of MSE will decreasing, which we have shown in the tables 1, 2, and figure 2 below:

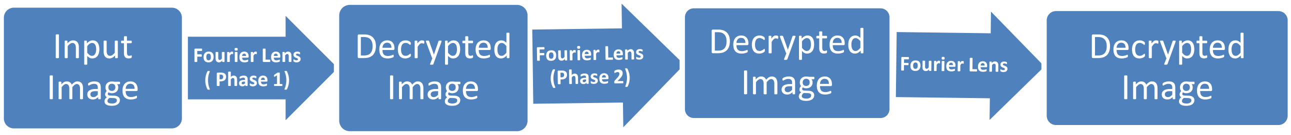

Figure 1: Schematic Diagram

Table 1: Computation of mean square error by changing the order of phase 1

Sr. No.	Fractional Order		Mean Square Error
	Phase 1	Phase 2	
1	0.05	-0.25	597.8093
2	0.10	-0.25	141.2637
3	0.15	-0.25	59.5178
4	0.20	-0.25	31.8424
5	0.25	-0.25	19.4559
6	0.30	-0.25	12.9538
7	0.35	-0.25	9.1684
8	0.40	-0.25	6.7991
9	0.50	-0.25	4.1604
10	0.60	-0.25	2.8472
11	0.70	-0.25	2.1387
12	0.80	-0.25	1.7484
13	0.90	-0.25	1.5504

Table 2: Computation of mean square error by changing the order of phase 2

Sr. No.	Fractional Order		Mean Square Error
	Phase 1	Phase 2	
1	0.25	-0.05	597.8093
2	0.25	-0.10	141.2637
3	0.25	-0.15	59.5178
4	0.25	-0.20	31.8424
5	0.25	-0.25	19.4559
6	0.25	-0.30	12.9538
7	0.25	-0.35	9.1684
8	0.25	-0.40	6.7991
9	0.25	-0.50	4.1604
10	0.25	-0.60	2.8472
11	0.25	-0.70	2.1387
12	0.25	-0.80	1.7484
13	0.25	-0.90	1.5504

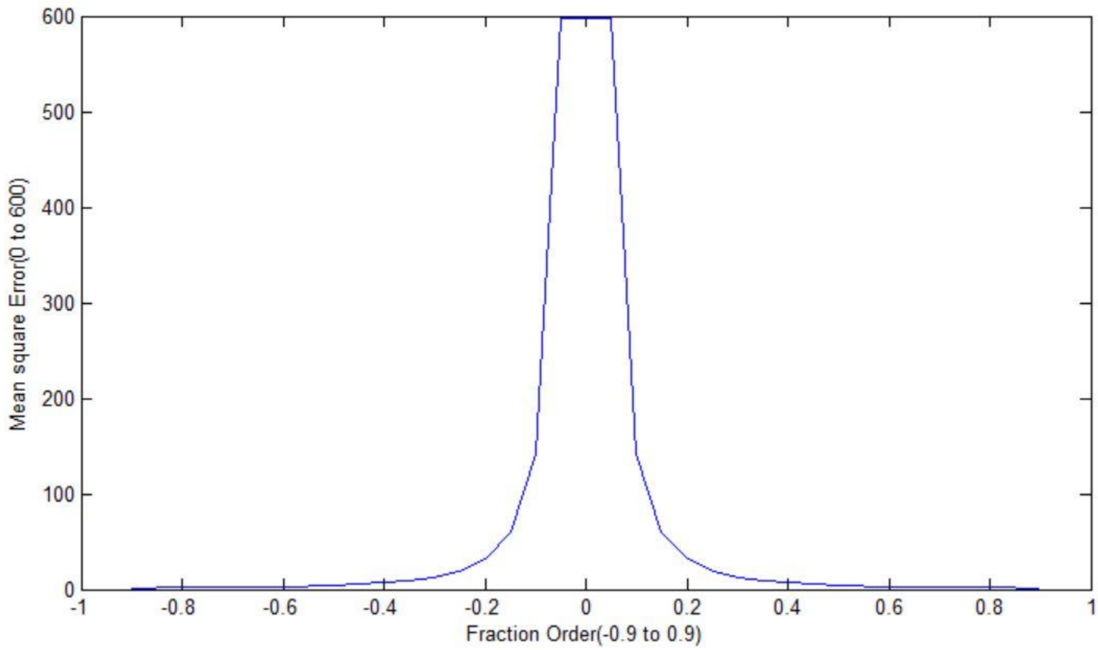

Figure 2: Mean square Error V/s Fraction Order Relation

4. Conclusion

In this paper, we studied the effect on the MSE of the image by FFT. We have found, using FFT that as we increase the value of fraction order, so decreasing the value of MSE. Due to this the quality of our image increases. Also, we observed from the simulation that Fraction order is inversely proportional to MSE.

References

1. Nisha Chittora and Deepika Babel, A Brief Study on Fourier Transform and its Applications, International Research Journal of Engineering and Technology, Vol. 05, No. 12, Dec 2018, pp. 1127–1131, [Online]. https://www.irjet.net/archives/V5/i12/IRJET-V5I12211.pdf.

2. J. V. Fischer, Four particular cases of the Fourier transform, Mathematics, Vol. 6, No. 12, 2018, pp. 4–7
doi: 10.3390/math6120335

3. Karan Asher, Fourier series And Fourier Transform, IOSR Journal of Mathematics, Vol. 4, Issue. 6, 2013, pp. 73–76.
doi: 10.9790/5728-0467376.

4. L. Zhong, L. Lichun, and L. Huiqi, Application Research on Sparse Fast Fourier Transform Algorithm in White Gaussian Noise, Procedia Comput. Sci., Vol. 107, No. Icict, 2017pp. 802–807.
doi: 10.1016/j.procs.2017.03.176.

5. H. C. Lin, Y. C. Ye, B. J. Huang, and J. L. Su, Bearing vibration detection and analysis using enhanced fast Fourier transform algorithm, Adv. Mech. Eng., Vol. 8, No. 10, 2016, pp. 1–14, ,
doi: 10.1177/1687814016675080.

6. G. G. Kumar, S. K. Sahoo, and P. K. Meher, 50 Years of FFT Algorithms and Applications, Circuits, Systems, and Signal Processing, Vol. 38, Issue. 12. 2019, pp. 5665-5698.

7. D. R. Jariwala and B. Patel, Accelerating Fast Fourier Transformation using Image Processing Techniques, Vol. 7, Issue. 4, 2017, pp. 10767–10771.

8. 8. T. Belabed, S. Jemmali, and C. Souani, FFT implementation and optimization on FPGA, 2018 Proceedings of the 4th International Conference on Advanced Technologies for Signal and Image Processing. ATSIP, Sousse,

Tunisia, March 21-24, 2018, pp. 1–6.
doi: 10.1109/ATSIP.2018.8364454.

9. Mitul Mehrotra, Geetika Pandey, Mandeep Singh Narula, Implementation of FFT Algorithm, International Journal of Engineering Sciences & Research Technology, Vol. 6, No. 5, 2017, pp.206–211.
doi:10.5281/zenodo.573508.

10. Y. Shechtman, A. Beck, and Y. C. Eldar, GESPAR: Efficient phase retrieval of sparse signals, IEEE Trans. Signal Process., Vol. 62, No. 4, 2014, pp. 928–938.
doi: 10.1109/TSP.2013.2297687.

11. S. Shenbaga Ezhil, Real-time application of Fourier transforms, Indones. J. Electr. Eng. Comput. Sci.,Vol. 8, No. 2, 2017, pp. 574–577.
doi: 10.11591/ijeecs.v8.i2.pp574-577.

12. Ashutosh and D. Sharma, Robust Technique for Image Encryption and Decryption Using Discrete Fractional Fourier Transform with Random Phase Masking, Procedia Technol., Vol. 10, 2013, pp. 707–714.
doi: 10.1016/j.protcy.2013.12.413

13. P. Poonam, R. Patil, P. Shankar, and A. Patil, Applications of Fourier Transform in Engineering Field, Vol. 7, No. 11, 2018, pp. 11090–11098.
doi: 10.15680/IJIRSET.2018.0711046

14. R. Tao, B. Deng, and Y. Wang, Research progress of the fractional Fourier transform in signal processing, Science in China Series F, Vol. 49, No. 1, 2006, pp. 1–25.
DOI: 10.1007/s11432-005-0240-y.

15. Y. Shang, S. Lei, and Y. Shang, Image encryption algorithm based on fractional Fourier transform, International Review on Computers and Software, Vol. 7, No. 5, 2012, pp. 1986–1989.

16. J. Kumar, P. Singh, A. K. Yadav, and A. Kumar, Asymmetric Cryptosystem for Phase Images in Fractional Fourier Domain Using LU-Decomposition and Arnold Transform, Procedia Computer Science, Vol. 132, 2018, pp. 1570–1577.
doi: 10.1016/j.procs.2018.05.121

17. N. Towghi, B. Javidi, and Z. Luo, Fully phase encrypted image processor, Journal of the Optical Society of America A, Vol. 16, No. 8, 1999, pp. 1915.
doi: 10.1364/josaa.16.001915.

18. U. Sara, M. Akter, and M. S. Uddin, Image Quality Assessment through FSIM, SSIM, MSE, and PSNR—A Comparative Study, Journal of Computer and Communications, Vol. 07, No. 03, 2019, pp. 8–18.
doi: 10.4236/jcc.2019.73002

19. Farida Memon, Mukhtiar Ali Unar, and Sheeraz Memon, Image Quality Assessment for Performance Evaluation of Focus Measure Operators, Mehran University Research Journal of Engineering and Technology, Vol. 34, No. 4, 2015, pp. 389–386.
http://publications.muet.edu.pk/research_papers/pdf/pdf1147.pdf.

20. P. Sharma, S. Sharma, and A. Goyal, An MSE (mean square error) based analysis of deconvolution techniques used for deblurring/restoration of MRI and CT Images, Proceedings of the Second International Conference on Information and Communicatin Technology for Competitive Strategies, ICTCS'16, Udaipur, India, March 4 - 5, 2016, pp.1-5.
doi: 10.1145/2905055.2905257

21. D. Asamoah, E. Ofori, S. Opoku, and J. Danso, Measuring the Performance of Image Contrast Enhancement Technique, International Journal of Computer Applications, Vol. 181, No. 22, 2018, pp. 6–13.
doi: 10.5120/ijca2018917899.