JNNCE Journal of Engineering & Management – A Peer Reviewed Bi-annual Journal
Available online @ https://jjem.jnnce.ac.in
ISSN: 2582-0079 (O)
https:// www.doi.org/10.37312/JJEM.2019.030204
Indexed in International Scientific Indexing (ISI)
Impact Factor Value: 1.025 for 2018-19
Volume: 3, Issue: 2
July - December 2019
Date of Publication: 12-30-2019

$(LCS)_n$-Manifold With Irrotational Curvature Tensors

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Abstract: *The object of the present paper is to study irrotational conharmonic, concircular, M-projective and quasi-conformal curvature tensors on $(LCS)_n$-manifold.*

Key Words: Lorentzian manifold, irrotational, conharmonic curvature tensor, concircular curvature tensor, M-projective curvature tensor, quasi-conformal curvature tensor.

AMS Subject Classification: 53C05, 53C20, 53C25, 53C50, 53D10.

Introduction: In 1989, Matsumoto [5] introduced a manifold M with a Lorentzian almost paracontact structure (ϕ, ξ, η, g). Mihai and Rosca [6] defined the same concept independently and obtained several results on this manifold. The author [9] introduced Lorentzian almost paracontact manifold with a structure of the concircular type and such a manifold is said to be a $(LCS)_n$-manifold, which generalizes the notion of LP-Sasakian manifolds. The $(LCS)_n$-manifolds were studied with various curvature conditions by Venkatesha [14], Prakasha [8], Yadav [15], Shaikh et al. ([10,11,12,13]) and others.

Let M^n be a Lorentzian manifold admitting concircular vector field ξ (a unit time like) called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1. \qquad (1.1)$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that for

$$g(X, \xi) = \eta(X), \qquad (1.2)$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha[g(X, Y) + \eta(X)\eta(Y)] \quad (\alpha \neq 0) \qquad (1.3)$$

for all vector fields X and Y. Here ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non zero scalar function satisfying

$$(\nabla_X \alpha) = (X\alpha) = d\alpha(X) = \rho\eta(X), \qquad (1.4)$$

ρ being a certain scalar function.

If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \qquad (1.5)$$

then from (1.3) and (1.5) we have

$$\phi^2 X = X + \eta(X)\xi, \qquad (1.6)$$

from which it follows that ϕ is a symmetric $(1,1)$ tensor called the structure tensor of the manifold. Bagewadi et al. [1,3,2] have studied irrotational projective curvature tensor, quasi-conformal curvature tensor and D-conformal curvature tensor in K-contact, Kenmotsu and trans-Sasakian manifolds. Also the authors have proved that these manifolds are Einstein manifold.

The paper is organized as follows, section 1 and section 2 gives the brief introduction to $(LCS)_n$-manifold, the basic equations of $(LCS)_n$-manifold and definitions of η-Einstein and generalized η-Einstein manifolds. Section 3 deals with the study of irrotational conharmonic curvature tensor, where the Ricci tensor vanishes resulting (3.13), provided $(\alpha^2-\rho)\neq 0$. Further, section 4, 5 and 6 are devoted to the study of irrotational concircular, M-projective and quasi-conformal curvature tensors respectively.

$(LCS)_n$-manifold:

A differentiable manifold M of dimension n is called Lorentzian concircular structure manifold [briefly $(LCS)_n$-manifold] if it admits a $(1,1)$ tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfies

$$\eta(\xi)=-1,\quad g(X,\xi)=\eta(X), \tag{2.1}$$

$$\phi^2X=X+\eta(X)\xi, \tag{2.2}$$

$$g(\phi X,\phi Y)=g(X,Y)+\eta(X)\eta(Y), \tag{2.3}$$

$$\phi\xi=0,\quad \eta(\phi X)=0, \tag{2.4}$$

for all $X,Y\in TM$. Also in a $(LCS)_n$-manifold M^n, the following relations are satisfied [12]

$$\eta(R(X,Y)Z)=(\alpha^2-\rho)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)], \tag{2.5}$$

$$R(X,Y)\xi=(\alpha^2-\rho)[\eta(Y)X-\eta(X)Y], \tag{2.6}$$

$$R(X,\xi)Z=(\alpha^2-\rho)[\eta(Z)X-g(X,Z)\xi], \tag{2.7}$$

$$R(\xi,X)Y=(\alpha^2-\rho)[g(X,Y)\xi-\eta(Y)X], \tag{2.8}$$

$$R(\xi,X)\xi=(\alpha^2-\rho)[X+\eta(X)\xi], \tag{2.9}$$

$$S(X,\xi)=(n-1)(\alpha^2-\rho)\eta(X),$$

$$Q\xi=(n-1)(\alpha^2-\rho)\xi, \tag{2.10}$$

$$(\nabla_X\phi)(Y)=\alpha[g(X,Y)\xi+2\eta(X)\eta(Y)\xi+\eta(Y)X], \tag{2.11}$$

$$S(\phi X,\phi Y)=S(X,Y)+(n-1)(\alpha^2-\rho)\eta(X)\eta(Y), \tag{2.12}$$

where R, S and Q are the Riemannian curvature tensor, the Ricci curvature and the Ricci operator respectively.

A $(LCS)_n$-manifold M^n is said to be an η-Einstein manifold if it satisfies

$$S(U,V)=\alpha g(U,V)+\beta\eta(U)\eta(V), \tag{2.13}$$

for any vector fields U and V, where α and β are smooth functions on (M^n,g). If $\beta=0$ then η-Einstein manifold becomes Einstein manifold.

Next, from (2.13), we have

$$QU=\alpha U+\beta\eta(U)\xi, \tag{2.14}$$

where Q is the Ricci operator.

Again, contracting (2.14) with respect to U and using (2.1), we see that

$$r=n\alpha-\beta. \tag{2.15}$$

Now, substituting $X=\xi$ and $Y=\xi$ in (2.13) and making use of (2.1) and (2.10), we obtain

$$-\alpha+\beta=-(n-1)(\alpha^2-\rho). \tag{2.16}$$

Equating (2.15) and (2.16), we get

$$\alpha=\frac{r-(n-1)(\alpha^2-\rho)}{(n-1)} \tag{2.17}$$

and

$$\beta=\frac{r-n(n-1)(\alpha^2-\rho)}{(n-1)}. \tag{2.18}$$

A $(LCS)_n$ manifold M^n is said to be a generalized η-Einstein manifold [16] if the following condition holds

$$S(X,Y)=\lambda g(X,Y)+\mu\eta(X)\eta(Y)+\nu\,\Omega(X,Y),$$

for any vector fields X and Y, where λ,μ and ν are smooth functions and $\Omega(X,Y)=g(\phi X,Y)$.

Irrotational Conharmonic curvature tensor

Definition: The conharmonic curvature tensor [4] on $(LCS)_n$-manifold M of dimension n is defined as

$$N(X,Y)Z=R(X,Y)Z-\frac{1}{n-2}[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY], \tag{3.1}$$

for any vector fields X,Y and Z on M.

Definition: Let D be a Riemannian connection, then the rotation (Curl) of conharmonic curvature tensor N on a Riemannian manifold M^n is defined as

$$RotN = (D_U N)(X,Y)Z + (D_X N)(U,Y)Z + (D_Y N)(X,U)Z - (D_Z N)(X,Y)U \quad (3.2)$$

With the help of second Bianchi identity, we have

$$(D_U N)(X,Y)Z + (D_X N)(U,Y)Z + (D_Y N)(X,U)Z = 0. \quad (3.3)$$

In view of (3.3), (3.2) becomes

$$RotZ = -(D_Z N)(X,Y)U. \quad (3.4)$$

If the conharmonic curvature tensor is irrotational, then $curl\, N = 0$ and so by (3.4), we see that

$$(D_Z N)(X,Y)U = 0, \quad (3.5)$$

which can be expressed as

$$D_Z(N(X,Y)U) = N(D_Z X,Y)U + N(X,D_Z Y)U + N(X,Y)D_Z U. \quad (3.6)$$

By replacing U by ξ in (3.6), we get

$$D_Z(N(X,Y)\xi) = N(D_Z X,Y)\xi + N(X,D_Z Y)\xi + N(X,Y)D_Z\xi. \quad (3.7)$$

In (3.1), if we put $Z = \xi$ and using (2.6), (2.10) and (2.14), we have

$$N(X,Y)\xi = \gamma[\eta(Y)X - \eta(X)Y], \quad (3.8)$$

Where,

$$\gamma = -\frac{\alpha^2-\rho}{n-2}. \quad (3.9)$$

Applying (3.8) in (3.7) and using (1.3), we obtain

$$N(X,Y)\phi Z = \gamma[g(Y,Z)X - g(X,Z)Y + \eta(Z)\{\eta(Y)X - \eta(X)Y\}]. \quad (3.10)$$

Substituting Z by ϕZ in (3.10) and using (2.2), (2.4), one can get

$$N(X,Y)Z = \gamma[g(Y,\phi Z)X - g(X,\phi Z)Y - \eta(Z)\{\eta(Y)X - \eta(X)Y\}]. \quad (3.11)$$

Now, comparing (3.1) and (3.11), we see that

$$\gamma[g(Y,\phi Z)X - g(X,\phi Z)Y - \eta(Z)\{\eta(Y)X - \eta(X)Y\}] = R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]. \quad (3.12)$$

On contracting with respect to T in (3.12) and making use of (3.9), we finally obtain

$$\frac{(n-1)(\alpha^2-\rho)}{n-2}g(Y,Z) + \frac{(\alpha^2-\rho)}{n-2}\eta(Y)\eta(Z) - \frac{(\alpha^2-\rho)}{n-2}g(\phi Z,Y) = 0. \quad (3.13)$$

Thus, we can state the following:

Theorem: If an n-dimensional $(LCS)_n$-manifold satisfies irrotational conharmonic curvature tensor, then the Ricci tensor vanishes resulting (3.13), provided $(\alpha^2 - \rho) \neq 0$.

Irrotational Concircular curvature tensor

An interesting invariant of a concircular transformation is the concircular curvature tensor Z and is given by [17, 18]

$$Z = R - \frac{r}{n(n-1)}R_0, \quad (4.1)$$

Where $R_0 = g(X,Y)W - g(W,Y)X$. Here R and r denotes Riemannian curvature and scalar curvature respectively.

Definition: Let D be a Riemannian connection, then the rotation (Curl) of concircular curvature tensor Z on a Riemannian manifold M^n is defined as

$$RotZ = (D_V Z)(W,X)Y + (D_W Z)(V,X)Y + (D_X Z)(W,V)Y - (D_Y Z)(W,X)V. \quad (4.2)$$

By virtue of second Bianchi identity, we have

$$(D_V Z)(W,X)Y + (D_W Z)(V,X)Y + (D_X Z)(W,V)Y = 0. \quad (4.3)$$

From (4.3), (4.2) reduces to

$$RotZ = -(D_Y Z)(W,X)V. \quad (4.4)$$

If the concircular curvature tensor is irrotational, then $curl\, Z = 0$ and by (4.4), we get

$$(D_Y Z)(W,X)V = 0, \quad (4.5)$$

that can be written as

$$D_Y(Z(W,X)V) - Z(D_Y W,X)V + Z(W,D_Y X)V + Z(W,X)D_Y V. \quad (4.6)$$

By treating $V = \xi$ in (4.6), we have

$$D_Y(Z(W,X)\xi) = Z(D_Y W,X)\xi + Z(W,D_Y X)\xi + Z(W,X)D_Y\xi. \quad (4.7)$$

In (4.1) if we put $Y = \xi$ and using (2.1) and (2.6), we obtain

$$Z(W,X)\xi = \sigma[\eta(X)W - \eta(W)X], \quad (4.8)$$

where $\sigma = [(\alpha^2 - \rho) - \frac{r}{n(n-1)}]. \quad (4.9)$

By the use of (4.8) in (4.7) and using (1.3), we obtain

$$Z(W,X)\phi Y = \sigma[g(Y,X)W - g(Y,W)X + \eta(Y)\eta(X)W - \eta(Y)\eta(W)X]. \quad (4.10)$$

On substituting Y by ϕY and using (2.2), (2.4) and (4.8) in (4.10), we get

$$Z(W,X)Y = \sigma[g(\phi Y,X)W - g(\phi Y,W)X - \eta(Y)\eta(X)W + \eta(Y)\eta(W)X].$$
(4.11)

Comparing (4.1) and (4.11) leads to

$$\sigma[g(\phi Y,X)W - g(\phi Y,W)X - \eta(Y)\eta(X)W + \eta(Y)\eta(W)X]$$
$$= R(W,X)Y - \frac{r}{n(n-1)}[g(Y,W)X - g(X,W)Y]. \quad (4.12)$$

Contracting (4.12) with respect to W and using (4.9), one can get

$$S(X,Y) = \lambda_1 g(X,Y) + \mu_1\eta(X)\eta(Y) + \nu_1 g(\phi X,Y),$$
(4.13)

where

$$\lambda_1 \frac{(n-1)(\alpha^2-\rho)}{n}, \quad (4.14)$$

$$\mu_1 = -\frac{(n-1)^2(\alpha^2-\rho)}{n} \quad (4.15)$$

$$\nu_1 = \frac{(n-1)^2(\alpha^2-\rho)}{n}. \quad (4.16)$$

Thus, we can state

Theorem: Let M^n be a $(LCS)_n$-manifold in which concircular curvature tensor is irrotational then the manifold is generalized η-Einstein manifold.

Irrotational M-projective curvature tensor

In 1971, the authors [8] defined a tensor field W^* on a Riemannian manifold as follows

$$W^*(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY],$$
(5.1)

where R, S denotes respectively Riemannian curvature, Ricci tensor and Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$.

Definition: Let D be a Riemannian connection, then the rotation (Curl) of M-projective curvature tensor Z on a Riemannian manifold M^n is defined as

$$Rot W^* = (D_U W^*)(X,Y)Z + (D_X W^*)(U,Y)Z + (D_Y W^*)(X,U)Z - (D_Z W^*)(X,Y)U.$$
(5.2)

By virtue of second Bianchi identity, we have

$$(D_U W^*)(X,Y)Z + (D_X W^*)(U,Y)Z + (D_Y W^*)(X,U)Z = 0. \quad (5.3)$$

In view of (5.3), (5.2) becomes

$$Rot Z = -(D_Z W^*)(X,Y)U. \quad (5.4)$$

If the M-projective curvature tensor is irrotational, then $curl\ W^* = 0$ and by using (5.4), we see that

$$(D_Z W^*)(X,Y)U = 0, \quad (5.5)$$

which gives

$$D_Z(W^*(X,Y)U) = W^*(D_Z X,Y)U + W^*(X,D_Z Y)U + W^*(X,Y)D_Z U.$$
(5.6)

Taking $U = \xi$ in (5.6), we get

$$D_Z(W^*(X,Y)\xi) = W^*(D_Z X,Y)\xi + W^*(X,D_Z Y)\xi + W^*(X,Y)D_Z\xi.$$
(5.7)

Treating Z by ξ and using (2.6), (2.10) and (2.14) in (5.1), we obtain

$$W^*(X,Y)\xi = \tau[\eta(Y)X - \eta(X)Y], \quad (5.8)$$

where

$$\tau = \frac{1}{2}[(\alpha^2 - \rho) + \frac{1}{n-1}(\frac{r-(n-1)(\alpha^2-\rho)}{n-1})].$$

(5.9)

By applying (5.8) in (5.7) and using (1.3), we get

$$W^*(X,Y)\phi Z = \tau[g(Z,Y)X - g(Z,X)Y + \eta(Z)\{\eta(Y)X - \eta(X)Y\}].$$
(5.10)

Substituting Z by ϕZ and making use of (2.2), (2.4) and (5.8), the above equation yields

$$W^*(X,Y)Z = \tau[g(\phi Z,Y)X - g(\phi Z,X)Y - \eta(Z)\{\eta(Y)X - \eta(X)Y\}].$$
(5.11)

Equating (5.1) and (5.11), we get

$$\tau[g(\phi Z,Y)X - g(\phi Z,X)Y - \eta(Z)\{\eta(Y)X - \eta(X)Y\}] = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY].$$
(5.12)

On contracting, (5.12) yields

$$S(Y,Z) = \lambda_2 g(Y,Z) + \mu_2\eta(Y)\eta(Z) + \nu_2 g(\phi Z,Y), \quad (5.13)$$

where

$$\lambda_2 = \frac{(n-1)(\alpha^2-\rho)}{n},$$

$$\mu_2 = -\frac{(n-1)^2(\alpha^2-\rho)}{n} \text{ and}$$

$$\nu_2 = \frac{(n-1)^2(\alpha^2-\rho)}{n}.$$

Thus, we can state the theorem:

Theorem: If the M-projective curvature tensor on a $(LCS)_n$-manifold M^n is irrotational then the manifold is generalized η-Einstein manifold.

Irrotational Quasi-conformal curvature tensor

In 1968, Yano and Sawaki [19] defined and studied the concept of quasi-conformal curvature tensor $\tilde{C}$. According to the authors, quasi-conformal curvature tensor is given by

$$\tilde{C}(X,Y)Z = q_1 R(X,Y)Z + q_2[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{n}\left(\frac{q_1}{n-1} + 2q_2\right)[g(Y,Z)X - g(X,Z)Y], \quad (6.1)$$

where q_1 and q_2 are constants of which $q_2 \neq 0$.

Definition: Let D be a Riemannian connection, then the rotation (Curl) of quasi-conformal curvature tensor $\tilde{C}$ on a Riemannian manifold M^n is defined as

$$RotW^* = (D_U\tilde{C})(X,Y)Z + (D_X\tilde{C})(U,Y)Z + (D_Y\tilde{C})(X,U)Z - (D_Z\tilde{C})(X,Y)U. \quad (6.2)$$

With the help of second Bianchi identity, we have

$$(D_U\tilde{C})(X,Y)Z + (D_X\tilde{C})(U,Y)Z + (D_Y\tilde{C})(X,U)Z = 0. \quad (6.3)$$

In view of (6.3), (6.2) becomes

$$RotZ = -(D_Z\tilde{C})(X,Y)U. \quad (6.4)$$

If the quasi-conformal curvature tensor is irrotational, then $curl\ \tilde{C} = 0$ and by using (6.4), we obtain

$$(D_Z\tilde{C})(X,Y)U = 0, \quad (6.5)$$

which in turn gives

$$D_Z(\tilde{C}(X,Y)U) = \tilde{C}(D_ZX,Y)U + \tilde{C}(X,D_ZY)U + \tilde{C}(X,Y)D_ZU. \quad (6.6)$$

On substituting U by ξ in the above equation, we get

$$D_Z(\tilde{C}(X,Y)\xi) = \tilde{C}(D_ZX,Y)\xi + \tilde{C}(X,D_ZY)\xi + \tilde{C}(X,Y)D_Z\xi. \quad (6.7)$$

Treating $Z = \xi$ in (6.1) and making use of (2.1), (2.6), (2.10) and (2.14), we have

$$\tilde{C}(X,Y)\xi = \psi[\eta(Y)X - \eta(X)Y], \quad (6.8)$$

where

$$\psi = (q_1 + q_2(n-1))[\alpha^2 - \rho] - \left(\frac{q_1}{n-1} + 2q_2\right)\left[\frac{(n-1)(\alpha^2-\rho)}{n}\right]. \quad (6.9)$$

Now, applying (6.8) in (6.7) and by the use of (1.3), we see that

$$\tilde{C}(X,Y)\phi Z = \psi[g(Z,Y)X - g(Z,X)Y + \eta(z)(\eta(Y)X - \eta(X)Y)]. \quad (6.10)$$

By replacing Z by ϕZ in the above equation gives

$$\tilde{C}(X,Y)Z = \psi[g(\phi Z,Y)X - g(\phi Z,X)Y - \eta(Z)(\eta(Y)X - \eta(X)Y)]. \quad (6.11)$$

Comparing (6.1) and (6.11), we have

$$\psi[g(\phi Z,Y)X - g(\phi Z,X)Y - \eta(Z)(\eta(Y)X - \eta(X)Y)] = q_1R(X,Y)Z + q_2[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{n}\left(\frac{q_1}{n-1} + 2q_2\right)[g(Y,Z)X - g(X,Z)Y]. \quad (6.12)$$

Contracting the above expression, one can get

$$S(Y,Z) - \lambda_3\, g(Y,Z) + \mu_3\, \eta(Y)\eta(Z) + \nu_3\, g(\phi Z,Y), \quad (6.13)$$

where

$$\lambda_3 = \frac{(\alpha^2 - \rho)[q_1 + q_2(n-1)]}{n[q_1 + q_2(n-2)]},$$

$$\mu_3 = -\frac{(n-1)^2(\alpha^2-\rho)}{n}$$ and

$$\nu_3 = \frac{(n-1)^2(\alpha^2-\rho)}{n}.$$

Therefore, we can state:

Theorem: If a $(LCS)_n$-manifold M^n in which quasi-conformal curvature tensor is irrotational then the manifold is generalized η-Einstein manifold.

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