Higher-Dimensional Relativity and Scalar–Tensor Theories

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Abstract: We study the JBD Jordan-Brans–Dicke theories determine solutional behavior, stability analysis and asymptotic behavior of the same. We dilate upon a concatenated model in consideration to the fact that all the studies incorporated in the schedule are interrelated.

Keywords: Inflation field, Jordan-Brans–Dicke (JBD), Motion of composite bodies, Matter configuration,Coupling of universal scalar field, Yukaw's theory, Massive scalar field, Einstein gravity, Multi-black hole configuration, Psedoscalar.

I. Introduction

(1) Zee proposed the Higgs field of SM as scalar field and so the Higgs field to generate (eb) the gravitational constant.
(2) The interaction of the Higgs field with the particles that achieve mass through (e&eb) it is short-ranged (i.e. of Yukawa-type) and gravitational-like (one can get a Poisson equation from it), even within SM, so that (e) Zee's idea was taken 1992 for a scalar–tensor theory with Higgs field as (=) scalar field with Higgs mechanism.
(3) There, the massive scalar field couples to (e&eb) the masses, which are at the same time the source of (e) the scalar Higgs field, which generates (eb) the mass of the elementary particles through (e&eb) Symmetry Breakdown.
(4) These theories usually go through for vanishing scalar field to (e&eb) standard General Relativity and because of the nature of the massive field it is possible for such theories that the parameter of the scalar field (the coupling constant) do not have to be (=) as high as in standard JBD theories.
(5) However, it is not clear yet which of these models explains better neither the phenomenology found in nature nor if such scalar fields are (=) really given or necessary in nature.

2. Motivation behind research:

The model does not explain gravitation, although physical confirmation of a theoretical particle known as a graviton would account for it to a degree. Though it addresses strong and electroweak interactions, the Standard Model does not consistently explain the canonical theory of gravitation, general relativity, in terms of quantum field theory. The reason for this is, among other things, that quantum field theories of gravity generally break down before reaching the Planck scale. As a consequence, we have no reliable theory for the very early universe. Some physicists consider it to be ad hoc and inelegant, requiring 19 numerical constants whose values are unrelated and arbitrary.[1] Although the Standard Model, as it now stands, can explain why neutrinos have masses, the specifics of neutrino mass are still unclear. It is believed that explaining neutrino mass will require an additional 7 or 8 constants, which are also arbitrary parameters The Higgs mechanism gives rise to the hierarchy problem if some new physics (coupled to the Higgs) is present at high energy scales. In these cases, in order for the weak scale to be much smaller than the Planck scale, severe fine tuning of the parameters is required; there are, however, other scenarios that include quantum gravity in which such fine tuning can be avoided.[2] There are also issues of Quantum triviality, which suggests that it may not be possible to create a consistent quantum field theory involving elementary scalar particles. The model is inconsistent with the emerging "Standard Model of cosmology". More common contentions include the absence of an explanation in the Standard Model of particle physics for the observed amount of cold dark matter (CDM) and its contributions to dark energy, which are many orders of magnitude too large. It is also difficult to accommodate the observed predominance of matter over antimatter (matter/antimatter asymmetry). The isotropy and homogeneity of the visible universe over large distances seems to require a mechanism like cosmic inflation, which would also constitute an extension of the Standard Model. (F.J. Hasert; et al. (1974). "Observation of neutrino-like interactions without muon or electron in the Gargamelle neutrino experiment". Nuclear Physics B. 73 (1): 1. Bibcode:1974NuPhB..73....1H. doi:10.1016/0550-3213(74)90038-8., D. Haidt (4 October 2004). "The discovery of the weak neutral currents". CERN Courier., ^ D.J. Gross; F. Wilczek (1973). "Ultraviolet behavior of non-abelian gauge theories". Physical Review Letters. 30 (26): 1343–1346. Bibcode:1973PhRvL..30.1343G. doi:10.1103/PhysRevLett.30.1343.)

3. Variables used:

(1) JBD Jordan-Brans–Dicke theories (eb) although not changing the geodesic equation for test particles, change the motion of composite bodies to (e&eb) a more complex one.

(2) The coupling of a universal scalar field directly to (e&eb) the gravitational field gives rise to (eb) potentially observable effects for (e) the **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly.

(3) This is known as the **"Dicke–Nordtvedt" effect**, which leads to (eb) possible violations of the Strong as well as the Weak Equivalence Principle for extended masses.

(4) JBD-type theories with short-ranged scalar fields use (e) according to Yukawa's theory, massive scalar fields.

(5) The first of these theories was proposed by A. Zee 1979. He proposed a Broken-Symmetric Theory of Gravitation, combining the idea of Brans and Dicke with (e&eb) the one of **Symmetry Breakdown**, which is essential within (eb) the Standard Model SM of elementary particles, where the so-called Symmetry Breakdown leads to (e) mass generation as a consequence of particles interacting with (e&eb) the Higgs field.

(6) Zee proposed the Higgs field of SM as scalar field and so the Higgs field to generate (eb) the gravitational constant.

(7) The interaction of the Higgs field with the particles that achieve mass through (e&eb) it is short-ranged (i.e. of Yukawa-type) and gravitational-like (one can get a Poisson equation from it), even within SM, so that (e) Zee's idea was taken 1992 for a scalar–tensor theory with Higgs field as (=) scalar field with Higgs mechanism.

(8) There, the massive scalar field couples to (e&eb) the masses, which are at the same time the source of (e) the scalar Higgs field, which generates (eb) the mass of the elementary particles through (e&eb) Symmetry Breakdown.

(9) These theories usually go through for vanishing scalar field to (e&eb) standard General Relativity and because of the nature of the massive field it is possible for such theories that the parameter of the scalar field (the coupling constant) do not have to be (=) as high as in standard JBD theories.

(10) Though, it is not clear yet which of these models explains better neither the phenomenology found in nature nor if such scalar fields are (=) really given or necessary in nature.

(11) Nevertheless, JBD theories are used to explain (eb) **inflation** (for massless scalar fields then it is spoken of the **inflaton field**) after the **Big Bang as well as the quintessence**.

(12) Further, they are an option to explain dynamics usually given through the standard cold dark matter models, as well as MOND, Axions (from Breaking of a Symmetry, too), MACHOS,...

4. Notations:

Module One

G_{13} : Category one of JBD theories although not changing the geodesic equation for test particles

G_{14} : Category two of SAS

G_{15} : Category three of SAS

T_{13} : Category one of change the **motion of composite bodies** to (e&eb) a more complex one

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

motion of composite bodies to (e&eb) a more complex one

G_{16} : Category one of **motion of composite bodies ;** more complex one

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of more complex one; **motion of composite bodies**

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The coupling of a universal scalar field directly to (e&eb) the gravitational field gives rise to (eb) potentially observable effects for (e) the **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly

G_{20} : Category one of **coupling of a universal scalar field;** gravitational field

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of gravitational field ; **coupling of a universal scalar field**

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The coupling of a universal scalar field directly to the gravitational field gives rise to (eb) potentially observable effects for (e) the **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly

G_{24} : Category one of coupling of a universal scalar field directly to the gravitational field

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of potentially observable effects for (e) the **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The coupling of a universal scalar field directly to the gravitational field gives rise to potentially observable effects for (e) the **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly

G_{28} : Category one of **motion of matter configurations** to which gravitational energy contributes (e&eb) significantly

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of coupling of a universal scalar field directly to the gravitational field gives rise to potentially observable effects

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The coupling of a universal scalar field directly to the gravitational field gives rise to potentially observable effects for the motion of matter configurations to which gravitational energy contributes (e&eb) significantly

How significant is the contribution is given by the model

G_{32} : Category one of coupling of a universal scalar field directly to the gravitational field gives rise to potentially observable effects for the motion of matter configurations to which gravitational energy; significantly

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of significantly ; coupling of a universal scalar field directly to the gravitational field gives rise to potentially observable effects for the motion of matter configurations to which gravitational energy

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

This is known as the "Dicke–Nordtvedt" effect, which leads to (eb) possible violations of the Strong as well as the Weak Equivalence Principle for extended masses

G_{36} : Category one of "Dicke–Nordtvedt" effect

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of possible violations of the Strong as well as the Weak Equivalence Principle for extended masses

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

JBD-type theories with short-ranged scalar fields use (e) according to **Yukawa's theory, massive scalar fields**

G_{40} : Category one of JBD-type theories with short-ranged scalar fields; **Yukawa's theory, massive scalar fields**

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of **Yukawa's theory, massive scalar fields** ;JBD-type theories with short-ranged scalar fields

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The first of these theories was proposed by A. Zee 1979. He proposed a Broken-Symmetric Theory of Gravitation, combining the idea of Brans and Dicke with (e&eb) the one of **Symmetry Breakdown**, which is essential within (eb) the Standard Model SM of elementary particles, where the so-called Symmetry Breakdown leads to (e) mass generation as a consequence of particles interacting with (e&eb) the Higgs field.

G_{44} : Category one of **Broken-Symmetric Theory of Gravitation, combining the idea of Brans and Dicke; Symmetry Breakdown**, which is essential within (eb) the Standard Model SM of elementary particles, where the so-called Symmetry Breakdown leads to (e) mass generation as a consequence of particles interacting with (e&eb) the Higgs field

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of **Symmetry Breakdown**, which is essential within (eb) the Standard Model SM of elementary particles, where the so-called Symmetry Breakdown leads to (e) mass generation as a consequence of particles interacting with (e&eb) the Higgs field; **Broken-Symmetric Theory of Gravitation, combining the idea of Brans and Dicke**

T_{45} : Category two of SAS
T_{46} : Category three of SAS

ASYMPTOTIC STABILITY ANALYSIS (Results and Discussion)

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equil ibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)}, \quad \frac{\partial (b_i'')^{(9)}}{\partial G_j}(\,(G_{47})^*\,) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44.

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45}$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45}$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45}$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T_{46}^*\mathbb{G}_j\right)$$

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right) + \left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$$

$$+\left((\lambda)^{(2)} + (b_{18}')^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}')^{(2)} + (p_{18})^{(2)}\right) \left[\left(\left((\lambda)^{(2)} + (a_{16}')^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^*\right)\right]\left(\left((\lambda)^{(2)} + (b_{16}')^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^*\right)$$

$$+\left(\left((\lambda)^{(2)} + (a_{17}')^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^*\right)$$

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$$

$$+\left((\lambda)^{(2)} + (b_{18}')^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}')^{(2)} + (p_{18})^{(2)}\right) \left[\left(\left((\lambda)^{(2)} + (a_{16}')^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^* +\right.\right.$$

$(a_{17})^{(2)}(q_{16})^{(2)}G_{16}^*)\Big]\Big(((\lambda)^{(2)}+(b_{16}')^{(2)}-$
$(r_{16})^{(2)})s_{(17),(17)}T_{17}^*+(b_{17})^{(2)}s_{(16),(17)}T_{17}^*\Big)$
$+\Big(((\lambda)^{(2)}+(a_{17}')^{(2)}+(p_{17})^{(2)})(q_{16})^{(2)}G_{16}^*+$
$(a_{16})^{(2)}(q_{17})^{(2)}G_{17}^*\Big)\Big(((\lambda)^{(2)}+(b_{16}')^{(2)}-$
$(r_{16})^{(2)})s_{(17),(16)}T_{17}^*+(b_{17})^{(2)}s_{(16),(16)}T_{16}^*\Big)$
$\Big(((\lambda)^{(2)})^2+((a_{16}')^{(2)}+(a_{17}')^{(2)}+(p_{16})^{(2)}+$
$(p_{17})^{(2)})(\lambda)^{(2)}\Big)\ \Big(((\lambda)^{(2)})^2+((b_{16}')^{(2)}+$
$(b_{17}')^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)})(\lambda)^{(2)}\Big)$
$+\Big(((\lambda)^{(2)})^2+((a_{16}')^{(2)}+(a_{17}')^{(2)}+$
$(p_{16})^{(2)}+(p_{17})^{(2)})(\lambda)^{(2)}\Big)(q_{18})^{(2)}G_{18}$

$+((\lambda)^{(2)}+(a_{16}')^{(2)}+$
$(p_{16})^{(2)})\ ((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^*+$
$(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^*)$
$\Big(((\lambda)^{(2)}+(b_{16}')^{(2)}-$
$(r_{16})^{(2)})s_{(17),(18)}T_{17}^*+(b_{17})^{(2)}s_{(16),(18)}T_{16}^*\Big)\}=0$
$+((\lambda)^{(3)}+(b_{22}')^{(3)}-(r_{22})^{(3)})\{((\lambda)^{(3)}+$
$(a_{22}')^{(3)}+(p_{22})^{(3)})\ \Big[\Big(((\lambda)^{(3)}+(a_{20}')^{(3)}+$
$(p_{20})^{(3)})(q_{21})^{(3)}G_{21}^*+$
$(a_{21})^{(3)}(q_{20})^{(3)}G_{20}^*\Big)\Big]\Big(((\lambda)^{(3)}+(b_{20}')^{(3)}-$
$(r_{20})^{(3)})s_{(21),(21)}T_{21}^*+(b_{21})^{(3)}s_{(20),(21)}T_{21}^*\Big)$
$+\Big(((\lambda)^{(3)}+(a_{21}')^{(3)}+(p_{21})^{(3)})(q_{20})^{(3)}G_{20}^*+$
$(a_{20})^{(3)}(q_{21})^{(1)}G_{21}^*\Big)\Big(((\lambda)^{(3)}+(b_{20}')^{(3)}-$
$(r_{20})^{(3)})s_{(21),(20)}T_{21}^*+$
$(b_{21})^{(3)}s_{(20),(20)}T_{20}^*\Big)\Big(((\lambda)^{(3)})^2+((a_{20}')^{(3)}+$
$(a_{21}')^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)})(\lambda)^{(3)}\Big)$
$\Big(((\lambda)^{(3)})^2+((b_{20}')^{(3)}+(b_{21}')^{(3)}-(r_{20})^{(3)}+$
$(r_{21})^{(3)})(\lambda)^{(3)}\Big)+\Big(((\lambda)^{(3)})^2+((a_{20}')^{(3)}+$
$(a_{21}')^{(3)}+(p_{20})^{(3)}+$
$(p_{21})^{(3)})(\lambda)^{(3)}\Big)(q_{22})^{(3)}G_{22}$

$+((\lambda)^{(3)}+(a_{20}')^{(3)}+$
$(p_{20})^{(3)})\ ((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^*+$
$(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^*)$
$\Big(((\lambda)^{(3)}+(b_{20}')^{(3)}$
$-(r_{20})^{(3)})s_{(21),(22)}T_{21}^*+(b_{21})^{(3)}s_{(20),(22)}T_{20}^*\Big)\}$
$=0$
$+((\lambda)^{(4)}+(b_{26}')^{(4)}-(r_{26})^{(4)})\{((\lambda)^{(4)}+$
$(a_{26}')^{(4)}+(p_{26})^{(4)})$
$\Big[\Big(((\lambda)^{(4)}+(a_{24}')^{(4)}+(p_{24})^{(4)})(q_{25})^{(4)}G_{25}^*+$
$(a_{25})^{(4)}(q_{24})^{(4)}G_{24}^*\Big)\Big]$
$\Big(((\lambda)^{(4)}+(b_{24}')^{(4)}-$
$(r_{24})^{(4)})s_{(25),(25)}T_{25}^*+(b_{25})^{(4)}s_{(24),(25)}T_{25}^*\Big)$

$+\Big(((\lambda)^{(4)}+(a_{25}')^{(4)}+(p_{25})^{(4)})(q_{24})^{(4)}G_{24}^*+$
$(a_{24})^{(4)}(q_{25})^{(4)}G_{25}^*\Big)$
$\Big(((\lambda)^{(4)}+(b_{24}')^{(4)}-(r_{24})^{(4)})s_{(25),(24)}T_{25}^*$
$+(b_{25})^{(4)}s_{(24),(24)}T_{24}^*\Big)$
$\Big(((\lambda)^{(4)})^2+((a_{24}')^{(4)}+(a_{25}')^{(4)}+(p_{24})^{(4)}+$
$(p_{25})^{(4)})(\lambda)^{(4)}\Big)\ \Big(((\lambda)^{(4)})^2+((b_{24}')^{(4)}+$
$(b_{25}')^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)})(\lambda)^{(4)}\Big)$
$+\Big(((\lambda)^{(4)})^2+((a_{24}')^{(4)}+(a_{25}')^{(4)}+$
$(p_{24})^{(4)}+(p_{25})^{(4)})(\lambda)^{(4)}\Big)(q_{26})^{(4)}G_{26}$

$+((\lambda)^{(4)}+(a_{24}')^{(4)}+$
$(p_{24})^{(4)})\ ((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^*+$
$(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^*)\ \Big(((\lambda)^{(4)}+$
$(b_{24}')^{(4)}-$
$(r_{24})^{(4)})s_{(25),(26)}T_{25}^*+(b_{25})^{(4)}s_{(24),(26)}T_{24}^*\Big)\}=0$
$+((\lambda)^{(5)}+(b_{30}')^{(5)}-(r_{30})^{(5)})\{((\lambda)^{(5)}+$
$(a_{30}')^{(5)}+(p_{30})^{(5)})\ \Big[\Big(((\lambda)^{(5)}+(a_{28}')^{(5)}+$
$(p_{28})^{(5)})(q_{29})^{(5)}G_{29}^*+$
$(a_{29})^{(5)}(q_{28})^{(5)}G_{28}^*\Big)\Big]\Big(((\lambda)^{(5)}+(b_{28}')^{(5)}-$
$(r_{28})^{(5)})s_{(29),(29)}T_{29}^*+(b_{29})^{(5)}s_{(28),(29)}T_{29}^*\Big)$
$+\Big(((\lambda)^{(5)}+(a_{29}')^{(5)}+(p_{29})^{(5)})(q_{28})^{(5)}G_{28}^*+$
$(a_{28})^{(5)}(q_{29})^{(5)}G_{29}^*\Big)\Big(((\lambda)^{(5)}+(b_{28}')^{(5)}-$
$(r_{28})^{(5)})s_{(29),(28)}T_{29}^*+$
$(b_{29})^{(5)}s_{(28),(28)}T_{28}^*\Big)\Big(((\lambda)^{(5)})^2+((a_{28}')^{(5)}+$
$(a_{29}')^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)})(\lambda)^{(5)}\Big)$
$\Big(((\lambda)^{(5)})^2+((b_{28}')^{(5)}+(b_{29}')^{(5)}-(r_{28})^{(5)}+$
$(r_{29})^{(5)})(\lambda)^{(5)}\Big)+\Big(((\lambda)^{(5)})^2+((a_{28}')^{(5)}+$
$(a_{29}')^{(5)}+(p_{28})^{(5)}+$
$(p_{29})^{(5)})(\lambda)^{(5)}\Big)(q_{30})^{(5)}G_{30}$

$+((\lambda)^{(5)}+(a_{28}')^{(5)}+$
$(p_{28})^{(5)})\ ((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^*+$
$(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^*)$
$\Big(((\lambda)^{(5)}+(b_{28}')^{(5)}$
$-(r_{28})^{(5)})s_{(29),(30)}T_{29}^*+(b_{29})^{(5)}s_{(28),(30)}T_{28}^*\Big)\}$
$=0$
$+((\lambda)^{(6)}+(b_{34}')^{(6)}-(r_{34})^{(6)})\{((\lambda)^{(6)}+$
$(a_{34}')^{(6)}+(p_{34})^{(6)})\ \Big[\Big(((\lambda)^{(6)}+(a_{32}')^{(6)}+$
$(p_{32})^{(6)})(q_{33})^{(6)}G_{33}^*+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^*\Big)\Big]+$
$\Big(((\lambda)^{(6)}+(b_{32}')^{(6)}-(r_{32})^{(6)})s_{(33),(32)}T_{33}^*+$
$(b_{33})^{(6)}s_{(32),(32)}T_{32}^*\Big)$
$\Big(((\lambda)^{(6)})^2+((a_{32}')^{(6)}+(a_{33}')^{(6)}+(p_{32})^{(6)}+$
$(p_{33})^{(6)})(\lambda)^{(6)}\Big)\ \Big(((\lambda)^{(6)})^2+((b_{32}')^{(6)}+$
$(b_{33}')^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)})(\lambda)^{(6)}\Big)$

$+\left(\left((\lambda)^{(6)}\right)^2+\left((a_{32}')^{(6)}+(a_{33}')^{(6)}+\right.\right.$
$\left.\left.(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$
$+\left((\lambda)^{(6)}+(a_{32}')^{(6)}\right.$
$\left.+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right.$
$\left.+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$
$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-\right.\right.$
$\left.\left.(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$
$+\left((\lambda)^{(7)}+(b_{38}')^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+\right.$
$\left.(a_{38}')^{(7)}+(p_{38})^{(7)}\right)\left[\left(\left((\lambda)^{(7)}+(a_{36}')^{(7)}+\right.\right.\right.$
$\left.\left.\left.(p_{36})^{(7)}\right)(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)\right]$
$\left(\left((\lambda)^{(7)}+(b_{36}')^{(7)}-\right.\right.$
$\left.\left.(r_{36})^{(7)}\right)s_{(37),(37)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(37)}T_{37}^{*}\right)$
$+\left(\left((\lambda)^{(7)}+(a_{37}')^{(7)}\right.\right.$
$\left.\left.+(p_{37})^{(7)}\right)(q_{36})^{(7)}G_{36}^{*}(a_{36})^{(7)}(q_{37})^{(7)}G_{37}^{*}\right)\left(\left((\lambda)^{(7)}\right.\right.$
$\left.+(b_{36}')^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T_{37}^{*}$
$\left.+(b_{37})^{(7)}s_{(36),(36)}T_{36}^{*}\right)$
$\left(\left((\lambda)^{(7)}\right)^2+\left((a_{36}')^{(7)}+(a_{37}')^{(7)}+(p_{36})^{(7)}+\right.\right.$
$\left.\left.(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$
$\left(\left((\lambda)^{(7)}\right)^2+\left((b_{36}')^{(7)}+(b_{37}')^{(7)}-(r_{36})^{(7)}+\right.\right.$
$\left.\left.(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$
$+\left(\left((\lambda)^{(7)}\right)^2+\left((a_{36}')^{(7)}+(a_{37}')^{(7)}+\right.\right.$
$\left.\left.(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$

$+\left((\lambda)^{(7)}+(a_{36}')^{(7)}+\right.$
$\left.(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G_{37}^{*}+\right.$
$\left.(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)$
$\left(\left((\lambda)^{(7)}+(b_{36}')^{(7)}-\right.\right.$
$\left.\left.(r_{36})^{(7)}\right)s_{(37),(38)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(38)}T_{36}^{*}\right)\}=0$
$+\left((\lambda)^{(8)}+(b_{42}')^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+\right.$
$\left.(a_{42}')^{(8)}+(p_{42})^{(8)}\right)$
$\left[\left(\left((\lambda)^{(8)}+(a_{40}')^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G_{41}^{*}+\right.\right.$
$\left.\left.(a_{41})^{(8)}(q_{40})^{(8)}G_{40}^{*}\right)\right]$
$\left(\left((\lambda)^{(8)}+(b_{40}')^{(8)}-\right.\right.$
$\left.\left.(r_{40})^{(8)}\right)s_{(41),(41)}T_{41}^{*}+(b_{41})^{(8)}s_{(40),(41)}T_{41}^{*}\right)$
$+\left(\left((\lambda)^{(8)}+(a_{41}')^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G_{40}^{*}+\right.$
$\left.(a_{40})^{(8)}(q_{41})^{(8)}G_{41}^{*}\right)$
$\left(\left((\lambda)^{(8)}+(b_{40}')^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T_{41}^{*}+\right.$
$\left.(b_{41})^{(8)}s_{(40),(40)}T_{40}^{*}\right)$
$\left(\left((\lambda)^{(8)}\right)^2+\left((a_{40}')^{(8)}+(a_{41}')^{(8)}+(p_{40})^{(8)}+\right.\right.$
$\left.\left.(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$
$\left(\left((\lambda)^{(8)}\right)^2+\left((b_{40}')^{(8)}+(b_{41}')^{(8)}-(r_{40})^{(8)}+\right.\right.$
$\left.\left.(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$

$+\left(\left((\lambda)^{(8)}\right)^2+\left((a_{40}')^{(8)}+(a_{41}')^{(8)}+\right.\right.$
$\left.\left.(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$

$+\left((\lambda)^{(8)}+(a_{40}')^{(8)}+\right.$
$\left.(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G_{41}^{*}+\right.$
$\left.(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G_{40}^{*}\right)$
$\left(\left((\lambda)^{(8)}+(b_{40}')^{(8)}-\right.\right.$
$\left.\left.(r_{40})^{(8)}\right)s_{(41),(42)}T_{41}^{*}+(b_{41})^{(8)}s_{(40),(42)}T_{40}^{*}\right)\}=0$
$+\left((\lambda)^{(9)}+(b_{46}')^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+\right.$
$\left.(a_{46}')^{(9)}+(p_{46})^{(9)}\right)$
$\left[\left(\left((\lambda)^{(9)}+(a_{44}')^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G_{45}^{*}+\right.\right.$
$\left.\left.(a_{45})^{(9)}(q_{44})^{(9)}G_{44}^{*}\right)\right]$
$\left(\left((\lambda)^{(9)}+(b_{44}')^{(9)}-\right.\right.$
$\left.\left.(r_{44})^{(9)}\right)s_{(45),(45)}T_{45}^{*}+(b_{45})^{(9)}s_{(44),(45)}T_{45}^{*}\right)$
$+\left(\left((\lambda)^{(9)}+(a_{45}')^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G_{44}^{*}+\right.$
$\left.(a_{44})^{(9)}(q_{45})^{(9)}G_{45}^{*}\right)$
$\left(\left((\lambda)^{(9)}+(b_{44}')^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T_{45}^{*}+\right.$
$\left.(b_{45})^{(9)}s_{(44),(44)}T_{44}^{*}\right)$
$\left(\left((\lambda)^{(9)}\right)^2+\left((a_{44}')^{(9)}+(a_{45}')^{(9)}+(p_{44})^{(9)}+\right.\right.$
$\left.\left.(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$
$\left(\left((\lambda)^{(9)}\right)^2+\left((b_{44}')^{(9)}+(b_{45}')^{(9)}-(r_{44})^{(9)}+\right.\right.$
$\left.\left.(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$
$+\left(\left((\lambda)^{(9)}\right)^2+\left((a_{44}')^{(9)}+(a_{45}')^{(9)}+(p_{44})^{(9)}+\right.\right.$
$\left.\left.(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$

$+\left((\lambda)^{(9)}+(a_{44}')^{(9)}+\right.$
$\left.(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G_{45}^{*}+\right.$
$\left.(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G_{44}^{*}\right)$
$\left(\left((\lambda)^{(9)}+(b_{44}')^{(9)}\right.\right.$
$\left.\left.-(r_{44})^{(9)}\right)s_{(45),(46)}T_{45}^{*}+(b_{45})^{(9)}s_{(44),(46)}T_{44}^{*}\right)\}$
$=0$

$+\left((\lambda)^{(6)}+(b_{34}')^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+\right.$
$\left.(a_{34}')^{(6)}+(p_{34})^{(6)}\right)$
$\left[\left(\left((\lambda)^{(6)}+(a_{32}')^{(6)}+\right.\right.\right.$
$\left.\left.(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+\right.$
$\left.\left.(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$
$+$
$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-\right.\right.$
$\left.\left.(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$
$\left(\left((\lambda)^{(6)}\right)^2+\left((a_{32}')^{(6)}+(a_{33}')^{(6)}+\right.\right.$
$\left.\left.(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$
$\left(\left((\lambda)^{(6)}\right)^2+\left((b_{32}')^{(6)}+(b_{33}')^{(6)}-\right.\right.$
$\left.\left.(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$

$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$

$\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$
$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$

$+\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$
$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$
$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$
$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$

$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$
$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$
$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$
$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$

$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$
$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$

$+\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$
$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$
$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$
$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$

$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$
$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$
$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$
$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$

$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$
$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$

$+\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$
$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$
$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$
$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$

$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$
$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$
$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$
$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$
$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$
$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Practical applications:

1.Quark-Quark Interaction:

Quarks, leptons, hadrons, and bosons may seem exotic and esoteric but, in fact, they play a very mundane role in the world of radiation oncology. The familiar components of atomic nuclei, protons and neutrons (i.e., nucleons), are composed of smaller fundamental building blocks known as quarks. Quarks come in six “flavours,” which go by the somewhat capricious names of up, down, strange, charmed, bottom, and top, arranged from least to most massive. (The two most massive quarks, bottom and top, are also occasionally referred to as beauty and truth.) The six flavours of quarks can be arranged into three families (or “generations”): up and down, charmed and strange, and bottom and top (Table 1⇓)[3][4]. The proton is made up of two up quarks and one down quark while the neutron is composed of two down quarks and one up quark (Fig. 1⇓). From the net charges of the proton and neutron (+1 and 0, respectively), one can deduce that quarks must have fractional charges and that the charges are of opposite signs. The proton, with a net charge of +1, is composed of two up quarks, each with a charge of +2/3, and one down quark, with a −1/3 charge. The neutron, having no net electrical charge, is composed of two down quarks, each with a −1/3 charge, and one up quark with a +2/3 charge, cancelling out to a net of zero. Particles such as protons and neutrons, which are composed of three quarks, are classified as baryons from the Greek word for “heavy” (the same root as the relatively new field of bariatric medicine). Particles made up of a quark and antiquark pair are known as mesons. Negatively charged pi mesons, or pions, consist of pairs of down and antiup quarks and were formerly used in radiation therapy. James S. Welsh, M.S., M.D., UW Cancer Center, 410 Dewey Street, Wisconsin Rapids, Wisconsin 54494, USA.

2.Fermion- Boso Interaction Interaction

The basic Feynman rules for QCD can be discussed by considering the scattering between a quark and an anti-quark due to one-gluon exchange. To ease the discussion the flavours of the two quarks are assumed to be different. In this case, only a t-channel gluon-exchange has to be considered. A real boson can certainly turn into fermions. This is exactly what happens in pair production when a photon turns into an electron and positron. But you need to be very cautious when talking about virtual particles because virtual particles don't exist. The Feynman diagrams that we draw showing virtual particles are just a graphical representation of an integral called a propagator and do not show anything that actually happens. We can, and do, draw Feynman diagrams where a gauge boson turns into two fermions, but this is not showing a process that actually happens.(Physics Stack Exchange) When particles interact, it is essential that we know if they are considered as being identical or not. When bosons interact with bosons and fermions with fermions, they are considered identical particles and so they obey certain rules of Quantum Mechanics. More specifically, symmetric wave functions correspond to bosons and anti-symmetric wave functions correspond to fermions. A fermion and a boson is not considered as being identical particles. They are of different “kind”, so they need not obey these rules. Therefore, they are just like two quantum mechanical particles without the peculiarities that arise with being indistinguishable.(Quora)

Problems encountered in numerical analysis:

(1) This universal phenomenon has led to the prediction that frequent measurements during (e & eb) this non exponential period could inhibit (e) decay of the system, one form of the quantum Zeno effect.

(2) Subsequently, it was predicted that measurements applied more slowly could also enhance (eb+) decay rates, a phenomenon known as the quantum anti-Zeno effect.[5]

(3) In quantum mechanics, the interaction mentioned is called "measurement" because its result can be interpreted in terms of (e) classical mechanics.

(4) Frequent measurement prohibits (e) the transition.

(5) It can be a transition of a particle from one half-space to (e&eb) another (which could be used for (e) atomic mirror in an atomic nanoscope [6]) as in (eb) the time of arrival problem,[7][8] a transition of a photon in a waveguide from one mode to (e&eb) another, and it can be a transition of an atom from one quantum state to (e&eb) another.

(6) It can be a transition from the subspace without (e) decoherent loss of a qubit to (e&eb) a state with a qubit lost in a quantum computer.[9][10]

(7) In this sense, for (e) the qubit correction, it is sufficient to determine (eb) whether the decoherence has already occurred or not.

(8) All these can be considered as applications of the Zeno effect.[11] By its

nature, the effect appears only in (eb) systems with (e&eb) distinguishable quantum states, and hence is inapplicable to (e) classical phenomena and macroscopic bodies.

(9) The mathematician Robin Gandy recalled Alan Turing's formulation of the quantum Zeno effect in a letter to fellow mathematician Max Newman, shortly after Turing's death:

(10) [I]t is easy to show using standard theory that if a system starts in an eigenstate of some observable, and measurements are made of that observable N times a second, then, (eb) even if the state is not a stationary one, the probability that the system will be in (eb) the same state after, say, one second, tends to (e&eb) one as (e) N tends to infinity; that is, that continual observations will prevent (e) motion.

(11) Alan and I tackled one or two theoretical physicists with this, and they rather pooh-poohed it by saying that continual observation is (=) impossible.

(12) But there is nothing in the standard books (e.g., Dirac's) to this effect, so that at least the paradox shows up an inadequacy of Quantum Theory as usually presented. — Quoted by Andrew Hodges in Mathematical Logic, R. O. Gandy and C. E. M. Yates, eds. (Elsevier, 2001), p. 267.

(13) As a result of Turing's suggestion, the quantum Zeno effect is also sometimes known as the Turing paradox. The idea is implicit in the early work of John von Neumann on the mathematical foundations of quantum mechanics, and in particular the rule sometimes called the reduction postulate.[12]

(14) Unstable quantum systems are predicted to exhibit (eb) a short time deviation from (e) the exponential decay law.[13][14]

(15) It was later shown that the quantum Zeno effect of a single system is equivalent to the indetermination of the quantum state of a single system.[13][14][15]

Additional remarks and conjectures: The Higgs boson plays a unique role in the Standard Model, by explaining why the other elementary particles, except the photon and gluon, are massive. In particular, the Higgs boson explains why the photon has no mass, while the W and Z bosons are very heavy. Elementary-particle masses, and the differences between electromagnetism (mediated by the photon) and the weak force (mediated by the W and Z bosons), are critical to many aspects of the structure of microscopic (and hence macroscopic) matter. In electroweak theory, the Higgs boson generates the masses of the leptons (electron, muon, and tau) and quarks. As the Higgs boson is massive, it must interact with itself. Because the Higgs boson is a very massive particle and also decays almost immediately when created, only a very high-energy particle accelerator can observe and record it. Experiments to confirm and determine the nature of the Higgs boson using the Large Hadron Collider (LHC) at CERN began in early 2010 and were performed at Fermilab's Tevatron until its closure in late 2011. Mathematical consistency of the Standard Model requires that any mechanism capable of generating the masses of elementary particles becomes visible[clarification needed] at energies above 1.4 TeV;[12] therefore, the LHC (designed to collide two 7 TeV proton beams) was built to answer the question of whether the Higgs boson actually exists.[13] On 4 July 2012, two of the experiments at the LHC (ATLAS and CMS) both reported independently that they found a new particle with a mass of about 125 GeV/c2 (about 133 proton masses, on the order of 10×10−25 kg), which is "consistent with the Higgs boson".[14][15] It was later confirmed to be the searched-for Higgs boson.[3 (See Wikipedia for references)

Conclusion:

Positive nature of asymptotic solution has varied consequences in various fields of physics and mathematics. Equations can be used to uncover the phase structure of black hole solutions in higher-dimensional vacuum Einstein gravity, Kaluza-Klein black holes, i.e. static solutions with an event horizon in asymptotically flat spaces with compact directions, and stationary solutions with an event horizon in asymptotically flat space. Multi-black hole configurations on the cylinder and thin rotating black rings in dimensions higher than five can also be studied. .Phase diagrams can be drawn and wide ranging ramifications can be studied. Myers-Perry metric, which has spherical horizon topology, can be studied. Current bounds from (e) the polarization of the CMB predict (eb) the scale-invariant gravitational wave (GW) background of (e) inflation to be out of reach for (e) upcoming GW interferometers. prospect dramatically changes if the inflaton is (=) a pseudoscalar, in which case its generic coupling to (e&eb) any abelian gauge field provides (eb) a new source of GWs, directly related to (e&eb) the dynamics of inflation. This opens up (eb) new ways of probing (e&eb) the scalar potential responsible for (e) cosmic inflation. See Valerie Domcke, Mauro Pieroni, et al.

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