Impact of Applied Magnetic Field on Nonlinear Radiative Heat Transfer of Dusty Fluid over a Stretching Sheet

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Abstract**—**A detailed investigation of two-dimensional mixed convection flow of non-Newtonian fluid due to convectively heated stretching sheet in the presence of dust particles is conducted. To simulate the nonlinear thermal radiation effect, the Rosseland approximation has been used. The governing equations are transformed into a set of self similar nonlinear ordinary differential equations by means of suitable similarity transformations, which are then solved numerically by using Shooting method. The influence of pertinent parameters on the velocity and temperature distributions of both fluid and particle phases inside the boundary layer is observed and discussed in detail. Further, the features of skin friction coefficient and Nusselt number for different values of the governing parameters are also analyzed and discussed. Moreover, the numerical results of present study and those existing ones are tabulated for some limiting case.

Keyword: Dusty fluid,magnetic field, nonlinear radiation, stretching sheet.

I. INTRODUCTION

The study of nonlinear thermal radiative fluid flow passing over a stretched surface is an area of potential interest for the researchers due to its applications in many engineering and physical processes. Some applicable areas of such processes are solar power technology, nuclear plants, aircraft, propulsion devices, combustion chambers and chemical processes at high operating temperature. Particularly, the thermal radiation effect plays a vital role in controlling the heat transfer process at polymer processing industry. In view of this, Pantokratoras and Fang [1] were first to study the effect of nonlinear thermal radiation on the Sakiadis flow. Mushtaq et al [2] investigated the stagnation-point flow over a stretching sheet in the presence of nonlinear thermal radiation by utilizing upper-convected Maxwell fluid. They employed the numerical method to solve the nonlinear problem. Cortell [3] discussed the influence of thermal radiation effect on heat flow over a stretching sheet. He employed the nonlinear Rosseland approximation to incorporate the impact of thermal radiation. Mushtaq et al [4] presented the numerical solution for flow and heat transfer of nanofluid under nonlinear thermal radiation effect. Shehzad et al [5] examined the nonlinear thermal radiation effect on Jeffery nanofluid flow and they have modeled the three-dimensional flow of Jeffrey fluid over stretched surface and developed the series solutions to analyze the results. Hayat et al [6] produced an analytical solution for magnetohydrodynamic flow of nanofluid under velocity slip boundary and nonlinear thermal radiation. Hussain et al [7] analyzed the combined effect of linear and nonlinear radiation effect on stagnation point flow over a stretching surface. In this work, they solved the mathematical equations using similarity method. They have observed that the nonlinear thermal radiation offers better heat transfer rate at the sheet in comparison with linear thermal radiation case. Recently, the investigations by Animasaum et al [8], Krupalakshmi et al [9] and Mahanthesh et al [10] dealt with the flows under nonlinear radiation effect and different physical conditions.

The study of boundary layer flow over a stretching surface plays a pivotal role in industrial and engineering processes, such as liquid films in condensation process, aerodynamics and polymers, aerodynamic extrusion of plastic sheets, glass fiber, artificial fibers, metal spinning, the cooling process of metallic plate in a cooling bath and glass, wire drawing, paper production, crystal growing, cable coating etc. Sakiadis [11] was the first who has investigated the boundary layer flow over a moving continuous solid surface. Tsou et al [12] studied the combined analytical and experimental study of flow and heat transfer in the boundary layer on a continuous moving surface. The heat and mass transfer characteristics of self-similar boundary layer flow induced by continuous stretched surface has been carried out by Magyari et al [13].Ishak et al [14] have initiated the mixed convection boundary layer flow of a viscous fluid over a stretching vertical sheet in its own plane. Thereafter, exhaustive amount of work has been carried out related to boundary layer flow and heat transfer (see[15-17]).

For many years, engineers and scientists are concerned with the flow of dusty fluids which have got many important applications in include soilerosion by natural winds and dust entrainment in a cloud during nuclear explosion, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, transport processes, shock waves, Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries, fluidized beds, combustion, gas cooling systems, centrifugal separation of matter from fluid. Saffman [18] studied the work by formulating the governing equations for the flow of dusty fluid. Some representative studies in this direction can be refereed to Refs. [19-20]. Vajravelu and Neyfeh [21] analyzed the hydromagnetic flow of a dusty fluid over a stretching sheet with the effect of suction. Palani and Ganesan [22] initiated the heat transfer effects on dusty gas flow past a semi-infinite inclined plate. Gireesha et al [23] investigated the effect of non-uniform source/sink on flow and heat transfer of dusty fluid over a stretching sheet. Further Ramesh et al [24] studied the magnetic field and non-uniform source/sink effects on stagnation point flow of a dusty fluid over a stretching sheet. Recent works of [25-28] have been done in area of dusty flow over a stretching sheet or plate under different effects and conditions.

The study of magnetic field has important applications in medicine, physics and engineering. Many industrial types of equipment, such as MHD generators, pumps, bearings and boundary layer control, are affected by the interaction between the electrically conducting fluid and magnetic field .For this reason, many researchers have considered the magnetic field effects on the fluid flow. Koshiba et al [29] discussed the influence of induced magnetic field on large-scale pulsed MHD generator. Abel et al [30] discussed the heat transfer in a liquid film over an unsteady stretching surface with external magnetic field. Singh and Kumar [31] initiated the effects of thermal radiation on mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation and heat generation/absorption. Mabood et al [32] studied the MHD non-Darcian convective flow past a stretching sheet in a micropolar fluid with radiation.

The main objective of the present investigation is to study the effect of nonlinear thermal radiation on boundary layer flow of dusty fluid over a stretching sheet in the presence of magnetic field. The method of solution involves similarity transformation which reduces the partial differential equations into a set of non-linear ordinary differential equations. These non-linear ordinary differential equations have been solved by applying Runge-Kutta-Fehlberg forth-fifth order method (RKF45 Method). The velocity and temperature profiles for different values of flow parameters are presented in the figures. It is observed from all figures that the boundary conditions are satisfied asymptotically in all the cases which support the accuracy of numerical results. The numerical results of present investigation were compared with existing results and are found to be in good agreement.

II. MATHEMATICAL FORMULATION

Let us consider a steady flow of an incompressible dusty fluid over a stretching sheet. The flow is assumed to be confined to region of $y > 0$. The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow. A uniform magnetic field B_0 is imposed along the y-axis and the sheet being stretched with the velocity $u_w(x)$ along the x-axis. The number density is assumed to be constant and volume fraction of dust particles is neglected. The fluid and dust particles motions are coupled only through drag, heat transfer between them. The drag force is modeled using Stokes linear drag theory. Other interaction forces such as the virtual force the shear lift force and the spin-lift force will be neglected compared to the drag force. The terms T_W represent the temperature of the fluid at the sheet respectively. Whereas T_∞ denotes the ambient fluid temperature.

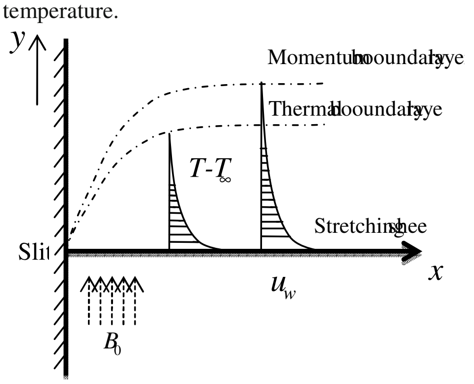

Figure 1: Physical model of the flow and coordinate system.

Under these conditions along with the boundary layer approximations, the governing boundary layer equations of momentum and energy for both the fluid phase and dust phase takes the following form[23];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{KN}{\rho}(u_p-u)-\frac{\sigma B_0^2}{\rho}u, \quad (2.2)$$

$$\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}=0, \quad (2.3)$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=KN(u-u_p), \quad (2.4)$$

Where (u,u_p) and (v,v_p) are the velocity components in x and y directions of the fluid and dust particle phase respectively. ρ and $\rho_p=mN$ are the density of the fluid and dust particle phase respectively. m and N are the mass and number density of the dust particles per unit volume respectively. σ is electrical conductivity of the fluid, μ is coefficient of viscosity of the fluid, $K=6\pi\mu r$ is Stokes drag coefficient and r is the radius of dust particle.

The appropriate boundary conditions applicable to the present problem are[23]:

$$u=u_w, v=0 \quad \text{at } y=0, \quad (2.5)$$
$$u\to 0,\ u_p\to 0,\ v_p\to v \text{ as } y\to\infty,$$

where $u_w(x)=bx$ is a stretching sheet velocity, $b>0$ is the stretching rate.

Equations (2.1) to (2.4) subjected to boundary conditions (2.5) admits self-similar solution in terms of the similarity function f, F and similarity variable η and they are defined as[23];

$$u=bxf'(\eta), \quad v=-\sqrt{\nu b}f(\eta), \quad \eta=\sqrt{\frac{b}{\nu}}y,$$
$$u_p=bxF'(\eta), \quad v_p=-\sqrt{\nu b}F(\eta), \quad (2.6)$$

where prime denotes the differentiation with respect to η. The equations (2.1) and (2.3) are identically satisfied, in terms of relations (2.6). In addition, equation (2.2) and (2.4) are reduced to following set of non-linear ordinary differential equations;

$$f'''(\eta)-[f'(\eta)]^2+f''(\eta)f(\eta)+l\,\beta_v[F'(\eta)-f'(\eta)]-Mf'(\eta)=0, \quad (2.7)$$

$$F(\eta)F''(\eta)-[F'(\eta)]^2+\beta_v[f'(\eta)-F'(\eta)]=0, \quad (2.8)$$

Transformed boundary conditions are;

$$f'=1,\ f(\eta)=0 \text{ at } \eta=0,$$
$$f'(\eta)\to 0\quad F'(\eta)\to 0,\quad F(\eta)\to f(\eta) \text{ as } \eta\to\infty, \quad (2.9)$$

where $l=\frac{Nm}{\rho}$ is the mass concentration parameter of dust particles, $M=\frac{\sigma B_0^2}{\rho b}$ is the magnetic parameter, $\tau_v=\frac{m}{K}$ is the relaxation time of the dust particles, i.e, the time required by a dust particle to adjust its velocity relative to the fluid, $\beta_v=\frac{1}{b\tau_v}$ is the fluid-particle interaction parameter.

III. HEAT TRANSFER ANALYSIS

The governing boundary layer heat transport equations for both fluid and dust phases are given by[23];

$$c_p\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\frac{\rho_p c_p}{\tau_T}(T_p-T)+\frac{\rho_p}{\tau_v}(u_p-u)^2-\frac{\partial q_r}{\partial y} \quad (3.1)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{c_p}{c_m\tau_T}(T_p-T), \quad (3.2)$$

where T and T_p are the temperature of the fluid and dust particles respectively, c_p and c_m are the specific heat of fluid and dust particles respectively, τ_T is the thermal equilibrium time i.e. the time required by the dust cloud to adjust its temperature to that of fluid, τ_v is the relaxation time of the dust particle i.e, the time required by a dust particle to adjust its velocity relative to the fluid, k is fluid thermal conductivity of the fluid and q_r is the radiative heat flux.

Using the Rosseland approximation for radiation, (2.5) radiation heat flux is simplified as

$$q_r=-\frac{16\sigma^*}{3k^*}T^3\frac{dT}{dy}, \quad (3.3)$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. T is the temperature across the boundary.

In view of the equation (3.3), the energy equation (3.1) becomes,

$$c_p\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{16\sigma^*}{3k^*}\left[T^3\frac{\partial^2 T}{\partial y^2}+3T^2\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\rho_p c_p}{\tau_T}(T_p-T)+\frac{\rho_p}{\tau_v}(u_p-u)^2, \quad (3.4)$$

Corresponding boundary conditions for the temperature are considered as follows[23];

$$T=T_w \text{ at } y=0$$
$$T\to T_\infty,\quad T_p\to T_\infty \text{ as } y\to\infty. \quad (3.5)$$

The dimensional fluid phase temperature $\theta(\eta)$ and dust phase temperature $\theta_p(\eta)$ are defined as:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty} \quad (3.6)$$

$$\theta''(\eta)+R\left[\left(1+(\theta_w-1)\theta(\eta)\right)^3\theta''(\eta)+3(\theta_w-1)\theta'^2(\eta)\left(1+(\theta_w-1)\theta(\eta)\right)^2\right]++Pr\,\theta'(\eta)f(\eta)+lPr\beta_t\left(\theta_p(\eta)-\theta(\eta)\right)+\beta_v l\,EcPr[F'(\eta)-f'(\eta)]^2=0$$

where

$$T=T_\infty(1+(\theta_w-1)\theta),\quad T_p=T_\infty(1+(\theta_w-1)\theta_p),\quad \theta_w=\frac{T_w}{T_\infty},\ \theta_w>1$$

being the temperature ratio parameter.

Using (3.6) into (3.2) and (3.4), we obtain the following non-linear ordinary differential equations: (3.7)

$$\theta_p'(\eta)F(\eta)-\gamma\beta_t[\theta_p(\eta)-\theta(\eta)]=0 \quad (3.8)$$

with

$\theta(\eta) = 1$ at $\eta = 0$, (3.9)

$\theta(\eta) \to 0, \quad \theta_p(\eta) \to 0$ as $\eta \to \infty$.

where the prime denote differentiation with respect to η, $Pr = \frac{\mu c_p}{k}$ is Prandtl number, $R = \frac{16\sigma^* T_\infty^3}{3k^* k}$ is the radiation parameter, $Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $\gamma = \frac{c_p}{c_m}$ is the specific heat ratio, $\beta_T = \frac{1}{b\tau_T}$ is the fluid-particle interaction parameter for temperature.

The physical quantities of interest like skin friction coefficient (C_f) and local Nusselt number (Nu_x) are defined as

$c_f = \frac{\tau_w}{\rho U_w^2}$ and $u_x = \frac{x q_w}{k(T_\infty - T_w)}$,

where the shear stress (τ_w) and surface heat flux (q_w) given by

$$\tau_w = \mu_0\left(\frac{\partial u}{\partial y}\right) \text{and} \, q_w = -k\frac{\partial T}{\partial y} + q_r.$$

Using the similarity transformations, we obtain,

$\sqrt{Re_x}C_f = f''(0)$ and

$\frac{1}{\sqrt{Re_x}}Nu_x = -[1 + R\theta_w^3]\theta'(0)$,

where $Re_x = \frac{x u_w(x)}{\nu}$ is the local Reynold's number

IV. NUMERICAL METHOD

The self-similar ordinary differential Equation(2.7)-(2.8) and (3.7)-(3.8) are highly non-linear in nature. Hence, they are solved numerically by using Runge–Kutta–Fehlberg fourth–fifth order method along with shooting approach. The Shooting technique attempts to make out suitable initial conditions for a relevant initial value problem that provides the solution to the original boundary value problem. This method is implemented in algebraic tool Maple called an algorithm 'shoot'. This algorithm is proven to be precise and accurate and it has been successfully used to solve a wide range of non-linear problems. In the numerical computation, in order to maintain accuracy, the step size $\Delta\eta$ in η and the location of the edge of the boundary layer η_∞ has adjusted for various values of parameters. In this study, a uniform grid of $\Delta\eta = 0.001$ was found to be satisfactory for the convergence criteria of 10^{-6}. On the other hand, the asymptotic boundary conditions at η_∞ are replaced by η_6, in such a way that the infinity boundary conditions are asymptotically achieved. For numerical results we considered the non-dimensional parameter values as $\theta_w = 1.2$, $Ec = 0.1$, $\gamma = 0.4$, $\beta_v = 0.6$, $\beta_t = 0.8$ $M = 0.5$, $Pr = 3$, $R = 0.6$, and $l = 0.5$. These values are kept as common in entire study except the variations in respective figures and tables.

V. RESULT AND DISCUSSION

The numerical investigation of the boundary layer flow and heat transfer over a stretching sheet in the presence of nonlinear thermalradiation and magnetic field is carried out. A parametric study is conducted in order to reveal some relevant physical aspects of the obtained results. The outcome of the parametric study is presented in the Figs. 2–8.

Figures 2-4 are sketched for the influence of magnetic parameter on velocity and temperature profiles for both fluid and dust phase respectively. Figure 2elucidates that, the velocity field and momentum boundary layer thickness of both phases reduces by increasing magnetic parameter. Further, the temperature profile of both fluid and dust phases enhanced with increasing magnetic parameter as shown in figures 3-4. Physically speaking, the magnetic field normal to an electrically conducting fluid has the tendency to produce a drag-like force called the Lorentz force, which acts in the direction opposite to that of the flow, causing a flow retardation effect. Reduction in the velocity is responsible for thickening the thermal boundary layer.

Figure 5 illustrate the effect of temperature ratio parameter (θ_w) on temperature profile for both fluid and dust phase. It is found that, the temperature profile and corresponding boundary layer thickness increases by increasing the temperature ratio parameter. It's clear that higher values of temperature ratio parameter improve the flow of the temperature profiles. This may happen because of increasing thermal conductivity of the flow.

The effect of Eckert number on temperature profile for both fluid and dust phases are respectively displayed in figures 6-7. One can observe that the temperature profile and the corresponding boundary layer thickness of both fluid and dust phases increases with an uplifting the values of Eckert number. This is because; the viscosity of the fluid takes energy from motion of the fluid and transforms it into the internal energy of the fluid which results in the heating of the fluid temperature. Hence the thermal boundary layer gets thicker with the increase in the viscous dissipation.

Figure 8 describes the impact of Prandtl number (Pr) over the temperature profile for dust phase. Hear, the thermal boundary layer thickness minimizes by maximizing the values of Prandtl number. The central reason for reduction in temperature is, a higher Prandtl number has relatively low thermal conductivity, which cause low heat penetration, thus the thermal boundary layer thickness of fluid and dust particles decreases with the rise in Prandtl number.

Table 1 shows the variation of skin friction coefficient and Nusselt number for different physical parameters in presence of linear and nonlinear radiation parameter. From the table it is observed that, Nusselt number is suppressed with nonlinearized thermal radiation when compared with linearized thermal radiation.Table2 displays

(3.9)

the variation of the skin friction coefficient and Nusselt number for different values of physical parameters. It is reported that, the Nusselt number is a decreasing function of Ec, γ, M and β_v but increase can be found for R, Pr, β_T, θ_w and l. Additionally, the skin friction coefficient increases for enhancing values of M, β_v and l.

VI. CONCLUSION

Anumerical analysis has been developed to investigate the effect of thermal stratification on boundary layer flow of a dusty fluid over a stretching surface embedded in a magnetic field in the presence of nonlinear thermal radiation. The governing boundary-layer equations for the problem are reduced to dimensionless ordinary differential equations by a suitable similarity transformation. Numerical computations for the effects of controlling parameters on velocity and temperature fields have been carried out. A comparison between the present numerical solutions and previously published results has been included, and the results are found to be in excellent agreement.The effects of various parameters on the flow and heat transfer are observed from the graphs, and are summarizedas follows:

- Temperature profile of both phases increases for increasing values of radiation parameter.
- An increase in the thermal boundary layer thickness and decrease in momentum boundary layer thickness was observed for the increasing values of the magnetic parameter M.
- Increase in the values of Ec, λ enhances the magnitude of local Nusselt number.
- Both velocity and temperature profiles are suppressed with linearized thermal radiation when compared with nonlinearizedthermal radiation.

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FIGURES AND TABLES

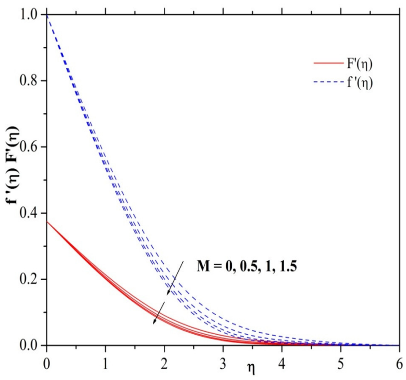

Figure 2: influence of M on $f'(\eta)$.

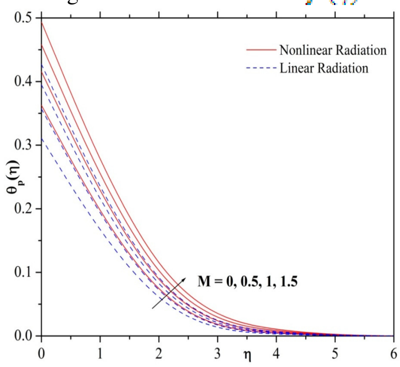

Figure 3: influence of M on $\theta_p(\eta)$.

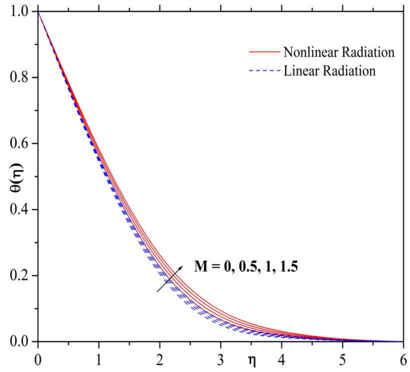

Figure 4: influence of M on $\theta(\eta)$.

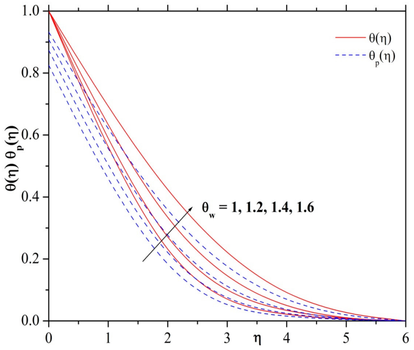

Figure 5: influence of θ_w on temperature profile.

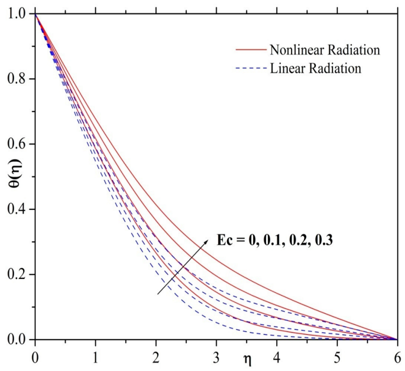

Figure 6: influence of Ec on $\theta(\eta)$.

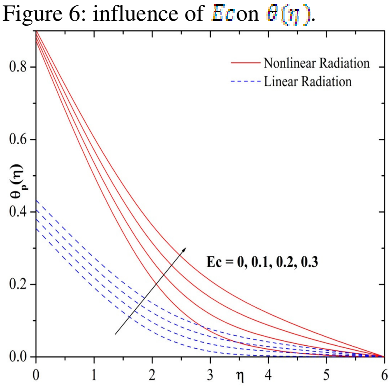

Figure 7: influence of Ec on $\theta_p(\eta)$.

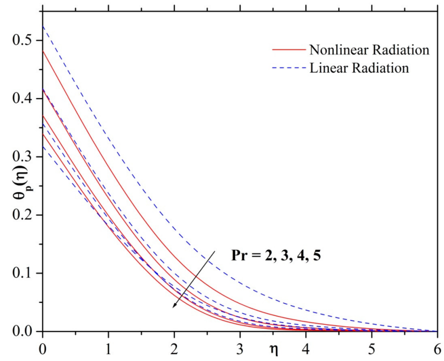

Figure 8: influence of Pr on $\theta_p(\eta)$.

Table 1 : Variation of skin friction coefficient and Nusselt number for different physical parameters in presence of linear and nonlinear radiation parameter.

Ec	Γ	M	Pr	R	β_v	β_t	l	Linear radiation $-\frac{1}{\sqrt{Re_x}}Nu_x$	Nonlinear radiation $-\frac{1}{\sqrt{Re_x}}Nu_x$
0								1.85173	1.64517
0.1								1.76199	1.56961
0.2								1.67226	1.49377
	1							1.72780	1.51943
	2							1.59894	1.39310
	3							1.53146	1.33033
		0						1.91179	1.70959
		0.5						1.84276	1.63763
		1						1.78456	1.57736
			2					1.42874	1.25940
			3					1.84276	1.63763
			4					2.19539	1.96116
				0				1.45739	0.98820
				0.5				1.78888	1.55315
				1				2.03172	1.92166
					0.5			1.84955	1.64406
					1			1.82028	1.61666
					1.5			1.79879	1.59708
						0.5		1.71038	1.51908
						1		1.91253	1.69848
						1.5		2.04635	1.81184
							0	1.37788	1.98960
							1	2.22278	2.56351
							2	2.84573	3.03726

Table 2:Variation of the skin friction coefficient and Nusselt number for different values of physical parameter

ε	θ_w	γ	M	Pr	R	β_v	β_t	l	$-\sqrt{Re_x}C_f$	$-\frac{1}{\sqrt{Re_x}}Nu_x$
0									1.29906	1.64517
0.1									1.29906	1.56961
0.2									1.29906	1.49377
	1								1.29906	1.84276
	1.2								1.29906	1.63763
	1.4								1.29906	1.56959
		1							1.29906	1.51943
		2							1.29906	1.39310
		3							1.29906	1.33033
			0						1.09001	1.70959
			0.5						1.29906	1.63763
			1						1.47902	1.57736
				2					1.29906	1.25940
				3					1.29906	1.63763
				4					1.29906	1.96116
					0				1.29906	0.98820
					0.5				1.29906	1.55315
					1				1.29906	1.92166
						0.5			1.29102	1.64406
						1			1.32289	1.61666
						1.5			1.34166	1.59708
							0.5		1.29906	1.51908
							1		1.29906	1.69848
							1.5		1.29906	1.81184
								0	1.36932	1.98960
								1	1.50001	2.56351
								2	1.62019	3.03726