Study on Shape Effects in Heat Transfer Phenomenon of Cu and Al_2O_3 Nanoparticle in The Presence of Nonlinear Thermal Radiation

B. J. Gireesha and M. Archana

Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta-577 451, Shimoga, Karnataka, INDIA.

***Abstract*— The phenomenon of heat transfer is scrutinized in the nanofluid flow problem by suspending Copper ( ) and Aluminium oxide (Al_2) nanoparticle with base fluid as water. The flow takes place over a stretching sheet along with the presence of nonlinear thermal radiation. The governing equations are simplified into ordinary differential equations with the help of bunch of similarity transformation. Reduced equations are solved numerically using Runge Kutta Fehlberg fourth fifth order technique. The influence of related parameters and their physical resultant interpretation are presented and discussed in detail with the help of graphical representation. It is reported that, the heat transfer augmentation is more for platelet shape nanoparticle than cylindrical shape. Moreover, heat transfer phenomena are more significant for nonlinear thermal radiation.**

***Index Terms*— Heat transfer, Nanofluid,Nonlinear thermal radiation, Shape effects.**

I. INTRODUCTION

Nanomaterials are being applied in more and more fields within engineering and technology. One of the key benefits of nanomaterials is that their properties differ from bulk material of the same composition. The properties of nanoparticles can be easily altered by varying their size, shape, and chemical environment. Among various nanoparticles, copper and aluminium nanoparticles has many potential applications in optical, biomedical, catalysis, cooling fluid or conductive inks and electronic fields due to their interesting properties like low cost preparation. Copper is found to be too soft for some applications, and hence it is often combined with other metals to form numerous alloys such as brass, which is a copper-zinc alloy while aluminium/aluminum nanoparticles have been widely researched and used, primarily because of their increased reactivity as compared with conventional micron-sized particle, high hardness, high stability, high insulation and transparency. The enhancement of the thermal conductivity of water in the presence of copper ( ) using the chemical reduction method was presented by Liu et al. [1]. Hwang et al. [2] have measured the pressure drop and convective heat transfer coefficient of water-based Al_2 nanofluids flow through a uniformly heated circular tube. Classical problem of forced convection boundary layer flow and heat transfer near the stagnation point on a permeable stretching/shrinking surface in a nanofluid was studied theoretically by Arifin et al. [3]. Hamad [4] has examined the convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field. A large number of experimental and theoretical studies have been carried out by many researchers on thermal conductivity of nanofluids with various effects [5-9]. Control Volume based finite element method was employed by Sheikholeslami et al. [10] to study the nanofluid flow by initiating the magnetic field dependent viscosity. The study of two dimensional boundary layer flow of a copper (Cu)-water nanofluid on a moving plate was investigated by Bakar et al. [11]. The effect of temperature and solid volume fraction on thermal conductivity of CNTs- Al_2/water nanofluids was discussed by Esfe et al. [12]. Rashidi et al. [13] have examined the impact of nanoparticle volume fraction and magnetic field in the nanofluid flow through the vertical channel consisting of sinusoidal walls.

Role of radiation heat transfer is superficial in many engineering processes which occur at high temperature. Linearized Rosseland approximation involves the dimensionless parameters called Radiation parameter and Prandtl number which are sustainable if the temperature difference between the plate and ambient fluid is small. But, for the larger temperature difference nonlinearized Rosseland approximation is valid. Mushtaq et al. [14] initiated the Rosseland approximation significance in the problem of two-dimensional stagnation-point flow of viscous fluid and analyzed the effect of nonlinear thermal radiation. Aaiza et al. [15] have concentrated on radiative heat transfer in mixed convection MHD flow of a different shapes of Al_2 in Ethylene Glycol based nanofluid in a channel filled with saturated porous medium. An analysis is carried out by Krishnamurthy et al. [16] to study the convective heat transfer of steady two-dimensional slip flow of nanofluid immersed in a porous medium over a stretching sheet in the presence of nonlinear thermal radiation. An emphasis has given on the impact of nonlinear thermal radiation and chemical reaction in two cases i.e., for constructive and destructive by Prasannakumara et al. [17]

for the slip flow of non-Newtonian nanofluid and they have determined that, increase of chemical reaction parameter for constructive case decreases the concentration profile whereas opposite behavior exist for the destructive case.

In this investigation, comparative study of (with Al_2 nanoparticle in different shape particularly - platelet and cylindrical shape in enhancing the rate of heat transfer has been analysed in the boundary layer flow and heat transfer of a nanofluid. Hamilton-Crosser model has been utilized. Further, nonlinear Rosseland approximation is employed. In the presence of suitable similarity transformations, the simplified governing two-dimensional equations are reduced. A well known numerical method - Runge-Kutta-Fehlberg fourth-fifth order along with shooting technique was incorporated to solve nonlinear ordinary differential equations. The existing various physical parameters are discussed in detail through graphical representation.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider the steady, laminar, incompressible two-dimensional boundary layer nano fluid flow over a stretching sheet (Fig. 1). The velocity of the stretching sheet is assumed to be $u_w(x) = ax$. Let u, v are the velocity components along the x, y directions respectively. Let T_w and C_w be the constant values of the temperature and nanoparticle concentration respectively and these constant values are assumed to be larger than the ambient temperature and nanoparticle concentration and are represented by T_∞ and C_∞ respectively. In the present flow problem, water is considered as a base fluid with suspended nanoparticles. Two types of nanoparticles- Cu and Al_2O_3 are contemplated in the flow problem. Nonlinear thermal radiation effects are incorporated in the energy equation.

Under the aforesaid assumptions, the governing basic equations for the present problem are given by,

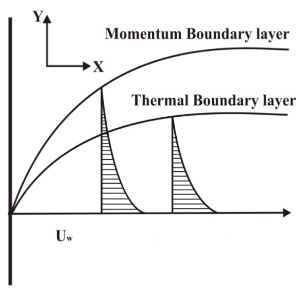

FIG 1. PHYSICAL MODEL AND CO-ORDINATE SYSTEM.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \quad (2.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}, \quad (2.3)$$

with the relevant boundary conditions,

$$\left.\begin{array}{l} u=ax, \quad v=0, \quad T=T_w, \quad C=C_w \ \text{at} \ y=0, \\ u\rightarrow 0, \ T\rightarrow T_\infty, \ C\rightarrow C_\infty \quad \text{as} \ y\rightarrow\infty, \end{array}\right\} \quad (2.4)$$

where T is the temperature of the nanofluid, μ_{nf} is the viscosity of the fluid, α_{nf} is the thermal diffusivity of the nanofluid, ρ_{nf} is the density of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid and q_r is the radiative heat flux.

In the present study, Hamilton and Crosser model for thermal conductivity and dynamic viscosity is incorporated, which is being valid for both spherical and non spherical shaped nanoparticles. According to this model:

$$\mu_{nf}=\mu_f(1+a\phi+b\phi^2), \quad (2.5)$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+(n-1)k_f+(n-1)(k_s-k_f)\phi}{k_s+(n-1)k_f-(k_s-k_f)\phi}. \quad (2.6)$$

In equation (2.3), the density ρ_{nf} and heat capacitance $(\rho c_p)_{nf}$ are defined as follows,

$$\left.\begin{array}{l} \rho_{nf} = (1-\phi)\rho_f+\phi\rho_p \\ (\rho C_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s \end{array}\right\}, \quad (2.7)$$

where ϕ is the nanoparticle volume fraction, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions respectively, ρ_f and ρ_s are the densities of the base fluid and solid nanoparticles, $(c_p)_s$ and $(c_p)_l$ are the specific heat capacities of solid nanoparticles and base fluids at constant pressure, and constants a and b are empirical shape factors and depend on the particle shape as given in Table 1. The n appearing in equation (2.6) is the empirical shape factor given by $n=\frac{3}{\Psi}$ where Ψ is the sphericity defined as the ratio between the surface area of the sphere and the surface area of the real particle with equal volumes. The values of Ψ for different shape particles are given in Table 1 and the thermophysical properties of water and nanoparticles are tabulated in table 2.

TABLE 1

Empirical shape factor and sphericity for different shapes of nanoparticle.

Model	Platelet	Blade	Cylinder	Brick
a	37.1	14.6	13.5	1.9
b	612.6	123.3	904.4	471.4
Ψ	0.52	0.36	0.62	0.81

TABLE 2

Thermophysical properties of water and nanoparticles

	$\rho(kg/m^3)$	$c_p(J/kgK)$	$k(W/m\text{-}K)$
$Water$	997.1	4179	0.613
Cu	8933	385	401
Al_2O_3	2701	902	239

Using the nonlinear Rosseland approximation for radiation, radiation heat flux q_r is simplified as,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = \frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial y}, \qquad (2.8)$$

where σ^* – Stefan-Boltzman constant and k^* - mean absorption coefficient.

The governing equations (2.1)-(2.3) subject to the boundary conditions (2.4) can be expressed in ordinary differential equation form by introducing the following similarity transformations,

$$u = axf'(\eta), v = -\sqrt{a\nu}f(\eta), \ \phi(\eta) = \frac{c-c_\infty}{c_w-c_\infty},$$
$$T = T_\infty(1+(\theta_w-1)\theta(\eta)), \eta = y\sqrt{\frac{a}{\nu}}. \qquad (2.9)$$

where η is the similarity variable, $\theta_w = \frac{T_w}{T_\infty}$, $\theta_w > 1$ being the temperature ratio parameter.

Employing the similarity variables, equations (2.2) and (2.3) are reduced to the following ordinary differential equations,

$$(1+a\phi+b\phi^2)f''' - \left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)[f'^2+ff''] = 0, \qquad (2.10)$$

$$\left[\frac{k_{nf}}{k_f} + Rd(1+(\theta_w-1)\theta)^3\right]\theta'' + Pr\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)f\theta' + 3R(1+(\theta_w-1)\theta)^2(\theta_w-1)\theta'^2 = 0, \qquad (2.11)$$

subjected to the boundary conditions (2.4) yields,

$$f=0, \ f'=1, \ \theta=1, \ \phi=1 \text{ at } \eta=0,$$
$$f'=0, \ \theta=0, \ \phi=0 \quad \text{as } \eta\to\infty. \qquad (2.12)$$

In the above equations, prime denotes differentiation with respect to η, Pr is the Prandtl number and Rd is the radiation parameter, which are defined respectively as,

$$Pr = \frac{\nu_f}{\alpha_f}, \quad R = \frac{16\sigma^* T_\infty^3}{3k^* k}.$$

The physical quantities of interest are the skin friction coefficient c_f and the Nusselt number Nu_x, which are defined as,

$$c_{fx} = \frac{\tau_w}{\rho_{nf}u_w^2}, \ Nu_x = \frac{x q_w}{k_{nf}(T_w-T_\infty)}, \qquad (2.13)$$

where the surface shear stress τ_w and the surface flux q_w are given by,

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = \left(-k_{nf}\frac{\partial T}{\partial y}+q_r\right)_{y=0}. \qquad (2.14)$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids respectively. Using the similarity variables of equation (2.9), we obtain,

$$\left.\begin{aligned}\sqrt{Re_x}c_{fx} &= \frac{(1+a\phi+b\phi^2)}{\left[1-\phi+\phi\left(\frac{\rho_p}{\rho_f}\right)\right]}f''(0),\\ \frac{Nu_x}{\sqrt{Re_x}} &= -\left(\frac{k_{nf}}{k_f}+R\theta_w^3\right)\theta'(0).\end{aligned}\right\} \qquad (2.15)$$

3. NUMERICAL PROCEDURE

The system of coupled nonlinear ordinary differential equations (2.10-2.11) along with the boundary conditions (2.12) is solved numerically using Runge–Kutta–Fehlberg fourth-fifth order method (RKF45 Method) along with shooting technique. The efficiency of this method is enhanced due to the reduction of computational time. With the help of shooting technique, missed initial conditions are inferred. We have considered infinity condition at a large but finite value of η where negligible variation in velocity, temperature and so on occurs. After fixing finite value for η_∞, integration is carried out with the help of Runge-Kutta-Fehlberg-45(RKF-45) method. This method has a procedure to determine an accurate solution if the proper step size h is being used. At each step, two different approximations for the solution are made and compared. If the two answers are in close agreement, the approximation is accepted otherwise the step size is reduced until to get the required accuracy. For the present problem, we took step size $\Delta\eta = 0.001, \eta_\infty = 5$ and accuracy to the fifth decimal place.

4. Results and Discussion

Numerical computations are carried out for several set of values of the radiation parameter (Rd), temperature ratio parameter (θ_w) and the nanoparticle volume fraction (ϕ) for two types of nanoparticle - Cu and Al_2O_3 having the platelet and cylindrical shape. In order to analyse the salient features of the problem, the numerical results are presented in the figures 2-11. These figures show the role of different shape and types of nanoparticle on heat transfer enhancement.

Figures 2 and 3 represent the variation of temperature component for the differing values of radiation parameter for Al_2O_3 and Cu nanoparticle respectively for platelet and cylindrical shape. It is observed that, temperature profile enhances for the larger values of radiation parameter. The reason for this is the presence of thermal radiation implies an enormous enlarging in the radiative heat which promotes the thermal state of the nanofluid creating its temperature to enhance.

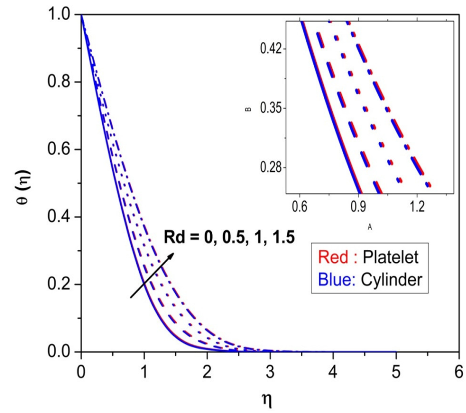

Figure-2: Impact of radiation parameter on temperature profile for Al_2O_3

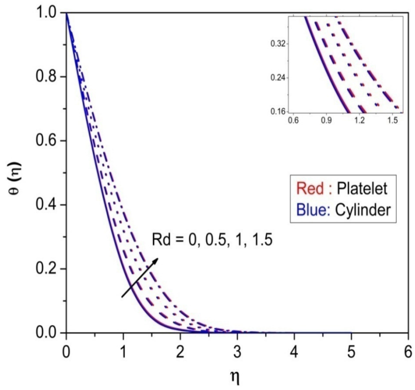

Figure-3: Impact of radiation parameter on temperature profile for C

Figures 4 and 5 respectively represent the influence of nanoparticle volume fraction on temperature profile for Al_2O_3 and Cu nanoparticle. It is found that, the effect of temperature profile increases for nanoparticle volume fraction. Physically, nanoparticles also serve to increase the thermal conductivity of the base fluid; as a result, heat is transferred from the sheet to the fluid with faster rate and consequently warms the thermal boundary layer region. Therefore, as the concentration of the nanoparticles in the base fluid increases, the temperature in the thermal boundary layer also increases.

Analysis of the figure 6 and 7 show that, the effect of the increasing values of the temperature ratio parameter is to enhance the temperature for both type of nanoparticles. This phenomenon occurs because it describes the thermal state of the fluid and with the increase of this parameter, temperature also increases.

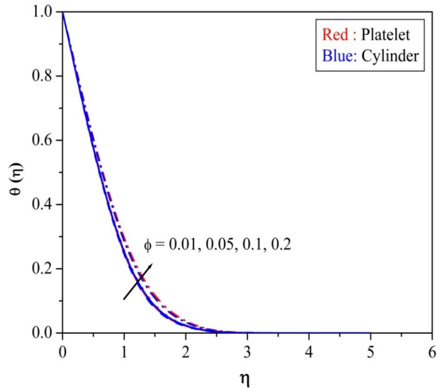

Figure-4: Impact of volume fraction on temperature profile for Al_2O_3

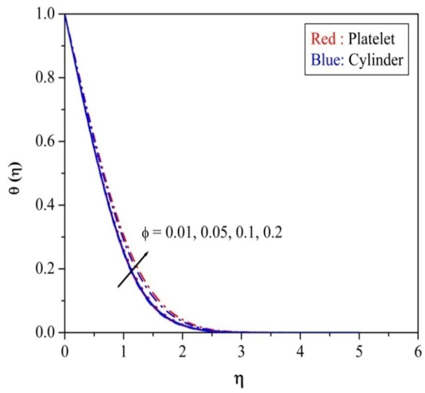

Figure-5: Impact of volume fraction on temperature profile for Cu

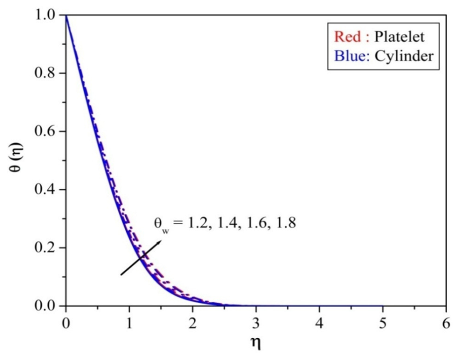

Figure-6: Impact of volume fraction on temperature profile for Al_2O_3

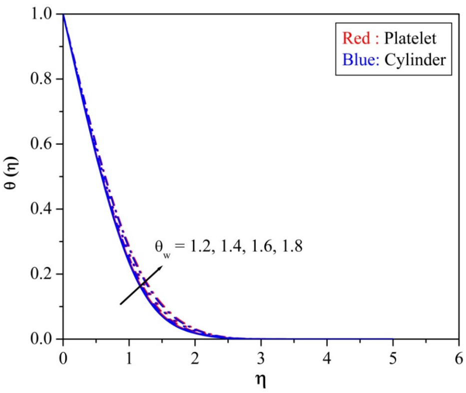

Figure-7: Impact of temperature ratio parameter on temperature profile for Cu

Figures 8 and 9 respectively represent the effect of radiation parameter for Al_2O_3 and Cu types of nanoparticle in two cases for platelet and cylindrical shape. It is evident from these figures that, the effect is found to be more significant in the Cu-water nanofluid than in the Al_2O_3-water for both shapes.

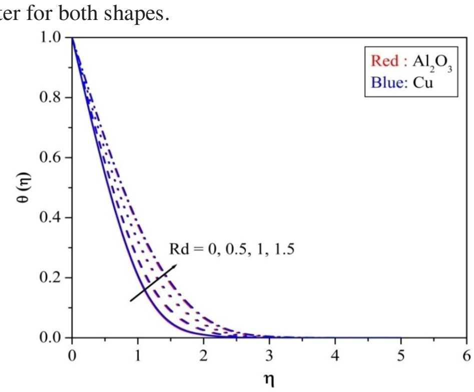

Figure-8: Impact of radiation parameter on temperature profile for platelet shape

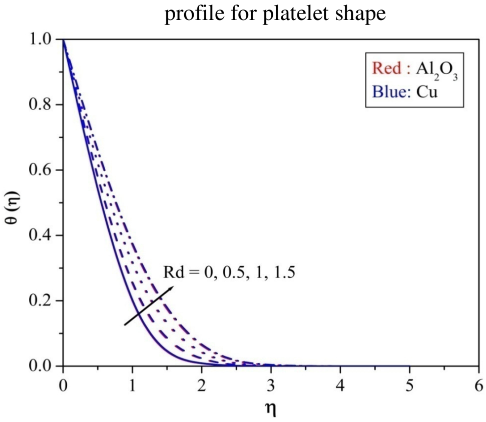

Figure-9: Impact of radiation parameter on temperature profile for cylindrical shape.

The effect of different particle shapes which are cylinder, platelet, brick and blade on the temperature of the nanofluid for both Cu and Al_2O_3 are is shown in figures 10 and 11 respectively. Besides, this effect is studied in the presence and absence of radiation parameter. It should be noted that, the effect of thermal conductivity increases with the increase of temperature but the viscosity decreases with the increase of temperature. It is clear that, elongated shape of nanoparticles like cylinder and platelet have minimum temperature because of the greater viscosity and thermal conductivity whereas blade has the highest temperature due to least viscosity and thermal conductivity.

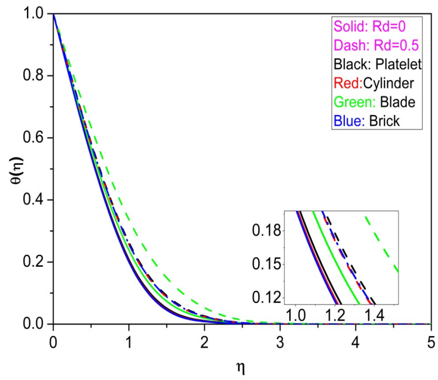

Figure-10: Impact of different nanoparticle shapes on temperature profile for Cu

The numerical values of Skin friction co-efficient and Nusselt number for various flow controlling parameters are tabulated in table 3 and 4 respectively for two shapes of the nanoparticle i.e. platelet and cylinder. For both platelet and cylindrical shaped nanoparticles, skin friction coefficient decreases for ϕ parameter and it is more for Cu nanoparticle. Further, Nusselt number increases for Rd, θ_w and ϕ for both shapes as well as for both nanoparticles.

TABLE 1

Empirical shape factor and sphericity for different shapes of nanoparticle.

Model	Platelet	Blade	Cylinder	Brick
a	37.1	14.6	13.5	1.9
b	612.6	123.3	904.4	471.4
Ψ	0.52	0.36	0.62	0.81

TABLE 2

Thermophysical properties of water and nanoparticles

	$\rho(kg/m^3)$	$c_p(J/kgK)$	$k(W/m\text{-}K)$
$Water$	997.1	4179	0.613
Cu	8933	385	401
Al_2O_3	2701	902	239

TABLE 3

Skin friction coefficient values for ϕ parameter

ϕ	For Cu		For Al_2O_3	
	Platelet	**Cylinder**	**Platelet**	**Cylinder**
0.01	0.8707	0.94044	0.85069	0.91873
0.05	0.57474	0.60474	0.52502	0.5518
0.1	0.42949	0.4206	0.37691	0.36977

TABLE 4

Numerical values of Nusselt Number for Rd, θ_w and ϕ parameter

Rd	θ_w	ϕ	For Cu		For Al_2O_3	
			Platelet	**Cylinder**	**Platelet**	**Cylinder**
0	1.5	0.1	1.54	1.59	1.57	1.62
.6			2.79	2.83	2.82	2.85
1			3.33	3.37	3.36	3.40
	1.2		2.23	2.28	1.90	2.30
	1.4		2.49	2.53	2.00	2.56
	1.6		2.77	2.81	2.13	2.83
		0.01	2.69	2.66	2.00	2.67
		0.05	2.708	2.715	2.079	2.739
		0.1	2.632	2.674	2.069	2.697

CONCLUSION

In this paper, the effects of radiative heat transfer in boundary layer flow of different shapes of Cu and Al_2O_3 nanoparticle in water based nanofluids was examined. Here, nonlinear thermal radiation is taken in to account. The influence of the different shapes of nanoparticles namely platelet and cylinder is determined. Variation of different parameters are analysed graphically and the corresponding results are explained as follows:

- Temperature profile increases for the increasing values of radiation parameter, temperature ratio parameter and volume fraction for both Al_2O_3 and Cu nanoparticles. Further, enhancement of temperature is little bit more for platelet shape of the nanoparticle as compared to cylindrical shape.
- Influence of Cu nanoparticle for enhancing the temperature is more as compared to Al_2O_3 nanoparticle.
- Blade shaped nanoparticle has highest temperature whereas cylinder and platelet shaped nanoparticle have lower temperature.

REFERENCES

Basic format for journals (when available online):

[1] Liu, Min-Sheng, Ching-Cheng Lin M, C.Y.Tsai, Chi-Chuan Wang. (2006, Aug). Enhancement of thermal conductivity with Cu for nanofluids using chemical reduction method. International Journal of Heat and Mass Transfer. **49**(17), pp. 3028-3033.

[2] Hwang, Sik K, S.P.Jang, S.U.S.Choi. (2009, Jan) Flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids in fully developed laminar flow regime. International journal of heat and mass transfer. **52**(1), pp. 193-199.

[3] N.M.Arifin, R.Nazar, I.Pop. (2011,) Viscous flow due to a permeable stretching/shrinking sheet in a nanofluid. Sains Malaysiana. **40**(12), pp. 1359-1367.

[4] M.A.A. Hamad. (2011,Apr). Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field. International communications in heat and mass transfer. **38**(4), pp. 487-492.

[5] M.G.Reddy. (2014, Jan). Influence of magnetohydrodynamic and thermal radiation boundary layer flow of a nanofluid past a stretching sheet. J. Sci. Res. **6**(2), pp. 257-272.

[6] F.Mabood, W.A. Khan, A. I.M. Ismail. (2015, Jan). MHD boundary layer flow and heat transfer of nanofluids over a non linear stretching sheet: A numerical study. J. Magnetism and Magnetic Materials. **374,** pp.569-576.

[7] Z. Abbas, M.Naveeda, M.Sajid. (2016, Mar). Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation. J. Mol. Liq. **215,** pp. 756-762.

[8] S.P.A.Devi, S.S.U.Devi. (2016,Jul). Numerical investigation of hydromagnetic hybrid $Cu - Al_2O_3$/water

nanofluid flow over a permeable stretching sheet with suction. Int. J. Nonlinear Sciences and Numerical Simulation. **17**(5), pp.249-257.

[9] P.H.Veena, S.M.Biradar, M.M.Nandeppanavar. (2016). Free convection boundary layer flow and heat transfer of a nano fluid over a moving plate with internal heat generation. Ind. Eng. Lett. **6**(2), pp.39-49.

[10] M.Sheikholeslami, H.B.Rokni. (2017,May). Magnetic nanofluid natural convection in the presence of thermal radiation considering variable viscosity. The European Physical Journal Plus. **132,** pp.1-12.

[11] N.A.A.Bakar, N.Bachok, N.Md.Arifin. (2016, Jun). Boundary layer flow and heat transfer on a moving plate in a Copper-Water nanofluid using Buongiorno Model. AIP Conf. Proc. 1739 020021 doi: 10.1063/1.4952501.

[12] Esfe, M.Hemmat, S.Saedodin, Wei-Mon Yan, M.Afrand, N. Sina. (2015, Oct). Study on thermal conductivity of water-based nanofluids with hybrid suspensions of $CNTs/Al_2O_3$ nanoparticles. Journal of Thermal Analysis and Calorimetry. **124**(1), pp. 455-460.

[13] M.M.Rashidi, M.Nasiri, M.Khezerloo, N.Laraqi. (2016, Mar) Numerical investigation of magnetic field effect on mixed convection heat transfer of nanofluid in a channel with sinusoidal walls, J. Magn. Magn. Mater. **93(1),** pp. 674-682. [14] A.Mushtaq, M.Mustafa, T.Hayat, A.Alsaedi. (2014, Jul). Nonlinear radiative heat transfer in the flow of nanofluid due to solar energy: A numerical study. J. Taiwan Inst. Chem. Eng.. **45**(4). pp 1176-1183.

[15] G.Aaiza, I.Khan, S.Shafie. (2015, Dec). Energy transfer in mixed convection MHD flow of nanofluid containing different shapes of nanoparticles in a channel filled with saturated porous medium. Nanoscale Research Letters. **10,** pp. 490.

[16] M.R.Krishnamurthy, B.C.Prasannakumara, R.S.R.Gorla, B.J.Gireesha. (2016, Aug). Non-Linear thermal radiation and slip effect on boundary layer flow and heat transfer of suspended nanoparticles over a stretching sheet embedded in porous medium with convective boundary conditions. Journal of Nanofluids. **5**(4), pp. 522-530.

[17] B.C.Prasannakumara, B.J.Gireesha, R.S.R.Gorla, M.R.Krishnamurthy. (2016, Sept) Effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium. Journal of Aerospace Engineering (American Society of Civil Engineers). **29**(5) 04016019.