RADIATION EFFECT ON MHD STAGNATION-POINT FLOW AND MELTING HEAT TRANSFER OF MICROPOLAR NANOFLUID OVER A LINEAR STRETCHING SHEET

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Abstract: This article investigates theradiation effects on the steady MHD boundary layer stagnation-point flow of a micropolar fluid towards a horizontal linearly stretching sheet in the presence of nanoparticles. A mathematical model is developed to study the heat transfer characteristics occurring during the melting process due to a stretching sheet.Suitable similarity transformations are employed to reduce thegoverning partial differential equations into the system of ordinary differential equations. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta-Fehlberg-45 method with Shootingtechnique. Results for the velocity, temperature and concentration distributions are presented graphically for different values of the pertinent parameters.

Key Words: Boundary layer flow, nanofluid, stretching sheet, melting heat transfer, stagnation point flow, thermal radiation.

1. Introduction

The boundary layer flow of non-Newtonian fluids has attracted a large class of applications in engineering practice, particularly in applied geophysics, geology, groundwater flow and oil reservoir engineering.In recent years, a great deal of interest has been generated in the area of two dimensionalboundary-layer flow over a stretching surface near a stagnation-point in view of its numerousand wide range of applications in various technical and industrial fields such as cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, heatexchangers placed in a low-velocity environment, solar central receivers exposed to windcurrents, and many hydrodynamic processes. Stagnation-point flow, describing the fluid motion near the stagnation region ofa circular body, exists for both the cases of a fixed or moving body in a fluid. The stagnationregion encounters the highest pressure, the heat transfer, and the rate of mass deposition.The two-dimensional flow of a fluid near astagnation point was first examined by Hiemenz [1]. Later, the problem of stagnation point flow was extended to various Newtonian/non-Newtonian fluids with various physical effects and different thermal boundary conditions. Following the pioneer work by Yang [2], the problem was extended to unsteady axis-symmetric stagnation-point flow by WilliamsIII [3] and to a general three dimensional stagnation-point flow by Jankowski and Gersting [4]. Very recently,Awaludin et al. [5] considered the stability analysis of stagnation point flow over a linearly stretching or shrinking sheet.Zaimi and Ishak [6]examined the effects of partial slip on stagnation-point flow and heat transfer due to a stretching vertical sheet.

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Over the last few years a considerable amount of experimental and numerical research has been

carried out to determine the role of natural convection in the kinetics of heat transfer accompanied with melting or solidificationeffect.Processes involving melting heat

transfer in non-Newtonian fluids have promisingapplications in thermal engineering such as oil extraction, magma solidification,melting of permafrost, geothermal energy recovery, silicon wafer process, thermalinsulation, etc.Roberts [7] was the first to describe the melting phenomena of iceplaced in a hot stream of air at a steady state. Epstein and Cho [8] studied, laminar filmcondensation on a vertical melting surface.Chamkha et al. [9] analyzed hydromagnetic, forcedconvection, boundary-layer flow with heat and mass transfer of a nanofluid over a horizontalstretching plate in the presence of a transverse magnetic field, melting and heat generationor absorption effects.Abel and JayashreeSanamani [10] carried out to study the steady two-dimensional stagnation-point flow and heat transfer from a warm, laminar liquid flow to a melting stretching sheet.

The theory of micropolar fluids takes into account themicroscopic effects arising from the local microstructure and intrinsic motion of the fluid elements, and is expected to provide a mathematical model for non-Newtonian fluid behavior.Theory of micropolar fluids was first proposed by Eringen[11].Bachok et al. [12] studied the effect of melting heat transfer in boundary layer stagnation-point flow towards astretching/shrinking sheet. Yacob et al. [13] investigated the boundarylayer stagnation-pointflow and heat transfer over a stretching/shrinking sheet immersedin a micropolar fluid in the presence of melting effect.Ishak et al. [14]obtained Dual solutions in mixed convection boundary layer flowof micropolar flow and heat transfer characteristics of micropolar fluid in the melting process towards a porous stretching/shrinking surface. Recently, Mohanty et al. [16]presented the the study of unsteady heat and mass transfer characteristics of a viscous incompressible electrically conducting micropolar fluid.

A Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes.Buongiorno [17] proposed a model for nanofluid. Goyal and Bhargava [18] theoretically investigated the conjugate effects of viscous dissipation and non-uniform heat source/ sink on the double-diffusive boundary layer flow of a viscoelastic nanofluid over a stretching sheet. Khan and Gorla [19] studied heat and mass transfer in non-Newtonian nanofluids over a stretching surface with prescribed wall temperature and surface nanoparticle concentration. The Buongiorno and Darcy models have been used by Khan et al [20] to investigate the effects of momentum slip on the double-diffusive free convective flow past a convectively heated vertical plate in a medium saturated with nanofluids.Kuznetsov and Nield[21] is the first who considered the problemon stretching sheet in nanofluids. Gorla et al[22] studied the effect of melting on heat transfer in a nanofluid flow past a permeable continuous moving surface.The model proposed by Tiwari and Das [23], is different from the above and which was also used by several authors (Abu- Nada [24], Muthtamilselvan et al. [25], Talebi et al. [26], Ahmad et al. [27], Bachok et al. [28, 29], Yacob et al. [30]). Buongiorno [17] studies the Brownian motion and the thermophoresis on the heat transfer characteristics with his model, while the model proposed by Tiwari and Das [23] analyzes the behavior of nanofluids taking into account the solid volume fraction. Mabood and Das [31] presented the MHD flow and melting heat transfer of a nanofluid over a stretching surface taking into account a second-order slip model and thermal radiation Hayat et al. [32] concentrated on the mathematical modeling for stagnation point flow and melting heat transfer of nanofluids over an impermeable stretching sheet with variable thickness.

In the recent year, non-Newtonian fluidshave become more and more important due to its industrial applications. Many studies are focused on non-Newtonian fluid as a base fluid with suspended nanoparticles over a stretching sheet.Rizwan et al. [33] presented the nanoparticles analysis for the Casson fluid model assuming the convective surface boundary conditions. Malik et al.[34] obtained the similarity solution for a steady boundary layer flow and heat transfer of a Cassonnanofluid over a vertical cylinder which is stretching exponentially along its radial direction. Jeffrey fluid has ability to exhibit the properties of ratio of stress relaxation to retardation and retardation. The steady flow of a Jeffrey fluid model in the presence of nano particles is studied by Nadeemet al.[35]. Shehzad et al. [36] developed a solar energy model to explore the characteristics of thermophoresis and Brownian motion in magnetohydrodynamic three-dimensional flowofnano Jeffrey fluid.In another investigation, Nadeemetal. [37] have examined a steady stagnation point flow of Jeffrey nanofluid over an exponential stretching surface under the boundary layer assumptions. Shehzad et al. [38] studied the influence of nanoparticles in MHD flow of Jeffrey fluid over a stretched surface. They considered the thermal and nanoparticles concentration convective boundary conditions. Hussain et al.[39] carried out an analysis to discuss Heat and mass transfer analysis of two-dimensional hydromagnetic flow of an incompressible Jeffrey nanofluid over an exponentially stretching surface.Khan et al.[40] analyzed the free convective boundary-layer flow of three-dimensional Oldroyd-B nanofluid flow over a bi directional stretching sheet with heat generation/absorption. They employed Oldroyd-B fluid model to describe rheological behavior of visco-elastic nanofluid. Hayatet al.[41] obtained series solution using a well-known analytic approach homotopy analysis method (HAM) for the flow of visco-elastic nanofluid over a stretching cylinder with simultaneous effects of heat and mass transfer.Nadeem andHussain [42] discussed thetwo-dimensional flow of Williamson fluid over a stretching-sheet under the effects of nano-sized particle also describedas nano Williamson fluid. Recently, Ramesh and Gireesha [43] reported heat source/sink effects on the steady boundary layer flow of a Maxwellfluid over a stretching sheet with convective boundary condition in the presence of nanoparticles. Hayat and Farooq [44] examined the stagnation point flow and melting heat transfer of Maxwell fluid over a stretching sheet with double-diffusive convection.Recently, Krishnamurthy et al. [45] investigated the radiation and chemical \reaction effects on the

steady boundary layer flow and melting heat transfer of MHD Williamson fluid through porous medium toward a horizontal linearly stretching sheet in the presence of nanoparticles.

The objective of the present study is to highlight the influence of melting heat transfer on the stagnation-point flow of a micropolar fluid in presence of Nanoparticle over a stretching sheet in presence of radiation and applied magnetic field. Such an analysis has not been yet reported even for any non-Newtonian nanofluid with or without stagnation-point.In the following sections,boundary layer equations for the flow and heat transfer of a micropolar fluid in thepresence of nanoparticles towards a horizontal stretching is given along with boundary conditions.Suitable similarity transformations are employed to reduce thegoverning partial differential equations into the system of ordinary differential equations.Numerical Procedure and numerical solutions of the equations of motion and heat are given.Detailed development of the flow structure and temperature profiles are presented graphically for different values of the pertinent parameters.

2. Mathematical formulation and solution of the problem

Consider a steady boundary layer flow and heat transfer of a micropolar fluid in the presence of nanoparticles towards a horizontal stretching sheet melting at a steady rate into a constant property, warm liquid of the same material. It is assumed that the velocity of the external flow is $U_e(x) = ax$ and the velocity of the stretching/shrinking sheetis $U_w(x) = cx$, where a is a positive constant, while $c$is a positive (stretching sheet) and x is the coordinate measured along the stretching sheet. It is also assumed that the temperature of the melting surface is T_m, while the temperature in the free-stream condition is T_∞, where $T_\infty > T_m$. The viscous dissipation and the heat generation or absorption are assumed to be negligible small.

Under these assumptions, the equations of motion and the equation representing temperature distribution in the Micropolar liquid flow in the presence of nanoparticle must obey the usual boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu+K}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + U_e\frac{dU_e}{dx} - \frac{\sigma B_0^2}{\rho}u, \qquad (2.2)$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right), \qquad (2.3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{D_{TC}}\left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}, \qquad (2.4)$$

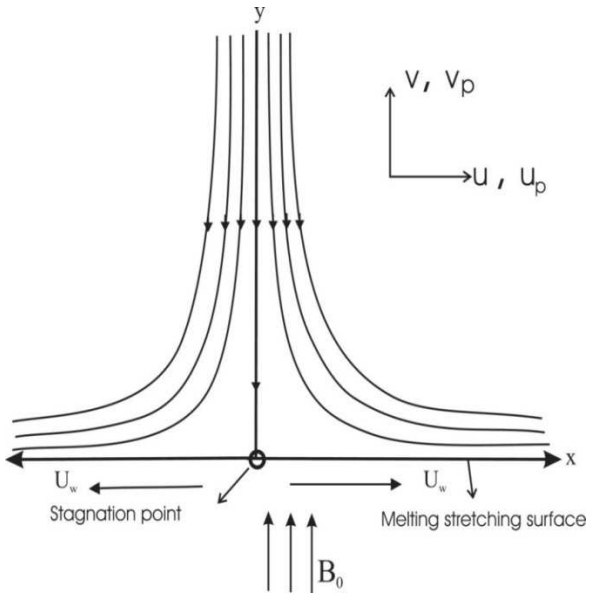

Figure 1: Schematic representation of boundary layer flow.

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{D_{TC}}\frac{\partial^2 T}{\partial y^2}. \qquad (2.5)$$

We assume that the boundary conditions of equations (2.1) − (2.5) are given by,

$$u = U_w(x) = cx, N = -n\frac{\partial u}{\partial y}, \quad T = T_m, \quad C = C_w \quad \text{at } y = 0,$$

$$u = U_e(x) = ax, \quad N = 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as } y \to \infty \text{ and} \qquad (2.6)$$

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho[\lambda + c_s(T_m - T_0)]v(x, 0). \qquad (2.7)$$

Here u and v are the velocity components along the x and y axes, respectively. Further, $\mu, K, \rho, T, j, N, \gamma$ and $\alpha_m$are respectively the dynamic viscosity, vortex viscosity (or the microrotationviscosity), fluid density, fluid temperature, microinertia density, microrotation (orangular velocity), spin gradient viscosity and nanofluid thermal diffusivity. Further, k is the thermalconductivity, λ is the latent heat of the fluid and c_s is the heat capacity of the solid surface.Equation (2.6) states that the heat conducted to the melting surface is equal to the heatof melting plus the sensible heat required to raise the solid temperature T_0 to its meltingtemperature T_m. The case $n = 0$, is called strong concentration indicates $N = 0$ near thewall, represents concentrated particle flows in which the microelements close to the wallsurface are unable to rotate. The case $n = \frac{1}{2}$ indicates the vanishing of anti-symmetricpart of the stress tensor and denotes weak concentrations. The case $n = 1$, is used for the modeling of turbulent boundary layer flows. Further, wefollow the work of many recent authors by assuming that $\gamma = \left(\mu + \frac{k}{2}\right)j = \mu\left(1 + \frac{R}{2}\right)j$, where $R = \frac{k}{\mu}$ is the micropolar or material parameter and $j = \frac{\nu}{a}$is a reference length.This assumption is invoked to allow the field of equations predicts the correct behavior inthe limiting case

when the microstructure effects become negligible and the total spin Nreduces to the angular velocity.

Using Rosseland approximation for radiation, the radiative heat flux is simplified as,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$$ (2.8)

where $\sigma^*$and k^* are the Stefan-Boltzmann constant and the mean absorption coefficientrespectively. The temperature differences within the flow are assumed to be sufficientlysmall so that T^4 may be expressed as a linear function of temperature T using a truncatedTaylor series about the free stream temperature T_∞ and neglecting the higher order terms,we get,

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4$$ (2.9)

The momentum, angular momentum, energy equations and concentration equations can betransformed into the corresponding ordinary differential equations by the following similarityvariables\

$$\psi = (a\nu)^{\frac{1}{2}} x f(\eta), \quad \eta = \left(\frac{a}{\nu}\right)^{1/2} y, \quad N = xa(a\nu)^{1/2} g(\eta),$$

$$\theta(\eta) = \frac{T-T_m}{T_\infty - T_m}, \quad \phi(\eta) = \frac{c-c_w}{c_\infty - c_w}.$$ (2.10)

The stream function ψ is defined such that

$$u = \frac{\partial \psi}{\partial y} \text{and } v = -\frac{\partial \psi}{\partial x}.$$

The transformed ordinary differential equations are,

$$(1+R)f''' + ff'' - (f')^2 + Rg' + 1 - Qf' = 0,$$ (2.11)

$$\left(1+\frac{R}{2}\right)g'' + fg' - f'g - R[2g + f''] = 0,$$ (2.12)

$$\left(\frac{1+Nr}{Pr}\right)\theta'' + f\theta' + Nb\phi'\theta' + Nt(\theta')^2 = 0,$$ (2.13)

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0,$$ (2.14)

where a prime denotes differentiation with respect to η, $Q = \frac{\sigma B^2 a}{\rho a}$is the magnetic parameter, $Pr = \frac{\nu}{\alpha_m}$is the Prandtl number, $Nr = \frac{16\sigma^* T_\infty^3}{3k^*}$ is the radiation parameter, $Nb = \frac{\tau D_B (c_\infty - c_w)}{\nu}$ is Brownian motion parameter, $Nt = \frac{\tau D_T (T_\infty - T_m)}{\nu T_\infty}$is the thermophoresis parameter and $Le = \frac{\nu}{D_B}$is the Lewis number.

The boundary conditions (2.6) become,

$$f'(0) = A, \quad g(0) = -nf''(0), \quad Prf(0) + M\theta'(0) = 0, \quad \theta(0) = 0, \quad \phi(0) = 0,$$

$$f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 1, \quad \phi(\infty) = 1,$$ (2.15)

where$A = \frac{c}{a}$is the stretching ($a > 0$) parameter and M is the dimensionless melting parameter

which is defined as,

$$M = \frac{c_f(T_\infty - T_m)}{\lambda + c_s(T_m - T_0)},$$ (2.16)

which is a combination of the Stefan number $\frac{c_f(T_\infty - T_m)}{\lambda}$and $\frac{c_s(T_m - T_0)}{\lambda}$ for the liquid and solidphases, respectively. It is worth mentioning that for $M = 0$ (melting is absent) and $R = 0$(viscous fluid) the problem reduces to that considered by Wang [46].

3. Numerical Procedure

The non-linear ordinary differential equations (2.11)-(2.14) with boundary conditions (2.15) have been solved using Runge-Kutta-Fehlberg fourth-fifth order method with Shootingtechnique.The non-linear ordinary differential equations (2.11)-(2.14) are of third order in f and second order in g, θ & $\phi$and these coupled differential equationsare first reduced into a

system of simultaneous ordinary equations.In order to solve this systemof equations using Runge-Kutta-Fehlbergfourth-fifth method, onerequire nine initial conditions. Butonly five initial conditions are known and however the values of $f'(\eta)$, $g(\eta)$, $\theta(\eta)$, $\phi(\eta)$ are known at$\eta = \infty$.These end conditions are used to obtain unknown initial condition$\eta = 0$ using Shooting technique.In Shootingmethod the boundary value problem (BVP)is reduced to an initial value problem (IVP).Theboundary value calculated is then matched with the real boundary value. Using trial anderror or some scientific approach, one attempts to get as close to the boundary value aspossible.The most essential step of this method is to choose the appropriate finite value for ηat∞.

We take infinity condition at a large but finite value of η where no considerable variation in velocity, temperature and so onoccur. We run our bulk computations with the value atηmax = 5, whichwas sufficient to achieve the far field boundary conditions asymptotically for all values ofthe parameters considered.

To have a check on the accuracy of the numerical procedureused, first test computations for $f''(0)$ and $\theta'(0)$ are carried out for regular fluid (Newtonian fluid) for various values of stretching parameter A and melting parameter M and compared with the available published results of Bachok et al. [3] and Yacob et al. [44] in table 1and they are found to be in excellent agreement.

4. Results and Discussion

In this section, a representative set of graphical results for velocity, temperature and nanoparticle volume fraction as well as the local Nusselt number and skin friction coefficient are presented and discussed for various parametric conditions.

Figures 2 reveals the velocity and temperature distributions for different values of melting parameter. It is observed that for increasing the values of M, the velocity and temperature distributions decreases. This is because of increase in M will increase the intensity of melting which act as blowing boundary condition at the stretching surface hence tends to thicken the boundary layer.Figures 3 exhibit the effect of magnetic parameter on velocity and temperature profiles. The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force, which opposes the flow. This resistive force tends to slow down the flow, so that the effect of increasing Q is to decrease the velocity and also causes decreases in its temperature.

Figure 4 explains the effect of Radiation parameter on temperature and nanoparticle fraction profiles. It is observed that, the temperature profile decreases and nanoparticle fraction profiles increases for increasing the values of Nr. This is because, the large values correspond to an increased dominance of conduction over radiation thereby decreasing buoyancy force and thickness of the thermal boundary layer.Figures 5 exhibit the velocity and angular velocity profiles for variable values of materialparameter (R). The fluid velocity and angular velocity is found to be decreases with increasing the values of R.It is because increasing the material parameterleads to the increase in the total viscosity of the flow, which thus retards the flow.

Figures 6 presents typical profile for temperature θ(η) and concentration ϕ(η) for various values of Thermophoretic parameter (Nt). It is observed that in both the cases an increase in the Nt leads to increase in fluid temperature and nanoparticle concentrations. This Thermophoresis serves to warm the boundary layer for low values of Pr and Lewis number (Le).The effect of Brownian motion parameter (Nb) on temperature and concentration are shown in figures 7. As expected, temperature in the boundary layer increases and the nanoparticle volume fraction decreases with the increase in Nb.This is because Brownian motion serves to warm the boundary layer and simultaneously exacerbates particle deposition away from the fluid regime or onto the surface, thereby accounting for the reduced concentration magnitudes.

Figures 8 indicates the effects of velocity and angular velocity profiles for n. From this,we observed that the velocity increases and angular velocity decreases for increasing thevalues of n.In figure 9 we exhibitsthe effect of A on velocity and angular velocity profiles.Both velocity and angular velocity profiles increases as the stretching ratio $A = a/c$ increases. This is due to the fact thata fixed value of $a/c > 1$ implies increase in the strainingmotion near the stagnation-point region resulting in increasedacceleration of the external stream. This leads to thinning of theboundary layer with increase in A.We noticed from the figure 10 that influence of stretching ratio A is to increase the temperature and concentration profile significantly.

The effect of Lewis number (Le)on concentration profile are illustrated in figure 11.It is clearly observed that the nanoparticles volume fraction as well as its boundary-layer thickness increase considerably as the Lewis number *Le* increases. Figure 12 depictsthe effect of Pr on temperature profiles respectively.In the presence of melting parameter, an increase in Prandtl number increases the temperature profiles and also from figure 13 we can see that the angular velocity increases near the plate and deceases far away from the plate for increasing values of Q.

The figures 14(a) and 14 (b) displays the nature of heat transfer coefficient against Nb for different valuesof Nt. Heat transfer increases for both Brownian motion parameter (Nb) and thermophoresisparameter (Nt).Similarly, the heat transfer decreases with Pr for different values of Nr. Figures 15(a) and 15(b)illustrate the variationsof skin friction $f''(0)$ verses material parameter R fordifferent values of melting parameter (M). One can be noticed that the skin friction decreaseswith an increase in the melting parameter (M). Similarly, the variation of skin friction $f''(0)$verses stretching parameter A for different values of n. From this it is observed that, theskin friction increases with an increase in the parameter n.

5. Conclusion

We have studied numerically the effect of melting on boundary layer stagnation flow of MicropolarNanofluid over a liner stretching sheet in presence of radiation and applied magnetic field. The model used for the nanofluidis the model proposed by Buongiorno which studies the effects of Brownian motion and the thermophoresis. Solutions of system of coupled equations are obtained using a Shooting method program with a Runge-Kutta-Fehlberg-45 method. Comparison with previously published work was performed and the results were found to be in good agreement.

The main findings of the study are summarized as follows;

- The boundary layer thickness and the velocity are found to be increases as the melting parameter increases, but the reverse trend is true for both the magneticand the radiation parameters.
- A boundary condition to account for melting was used at the interface between the solid and liquid phases.
- In the presence of a uniform magnetic field, increase inthe strength of the applied magnetic field decelerated thefluid motion along the wall of the plate

and also causes decreases in its temperature inside the boundary layer.

- The magnitudes of the skin friction coefficient and the local Nusselt number increase with an increase in the melting process.
- Angular velocity increases near the plate and deceases far away from the plate for increasing values of Q.

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Tables and Figures:

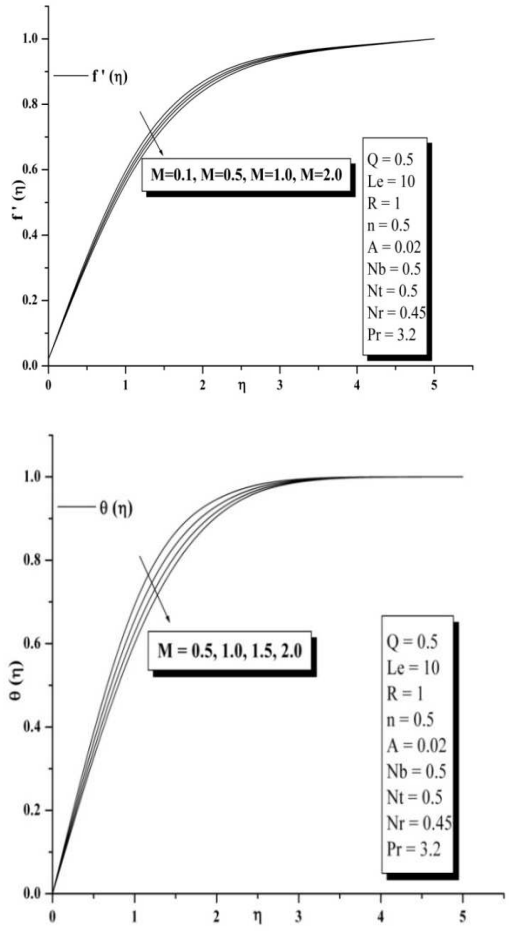
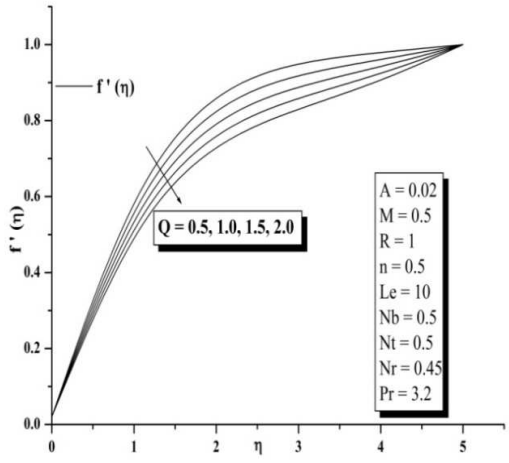


Figure 2: Effect of M on velocity and temperature profiles.



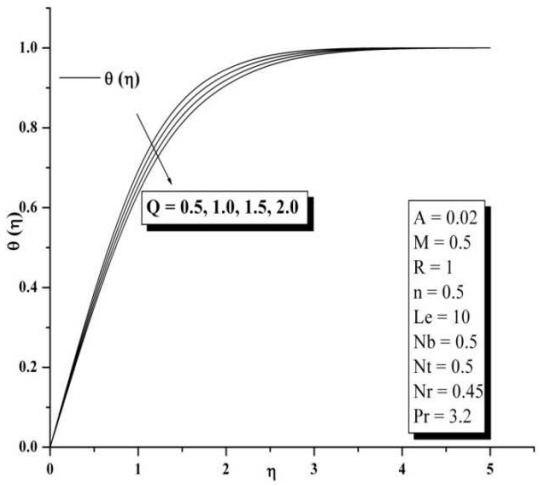


Figure 3: Effect of Q on velocity and temperature profiles.

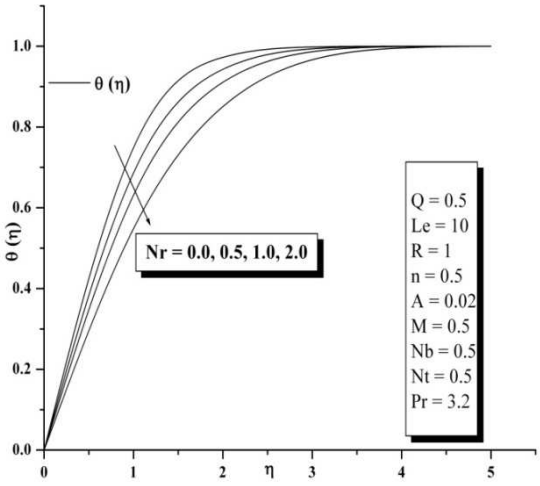


Figure 4: Effect of Nr on temperature and concentration profiles.

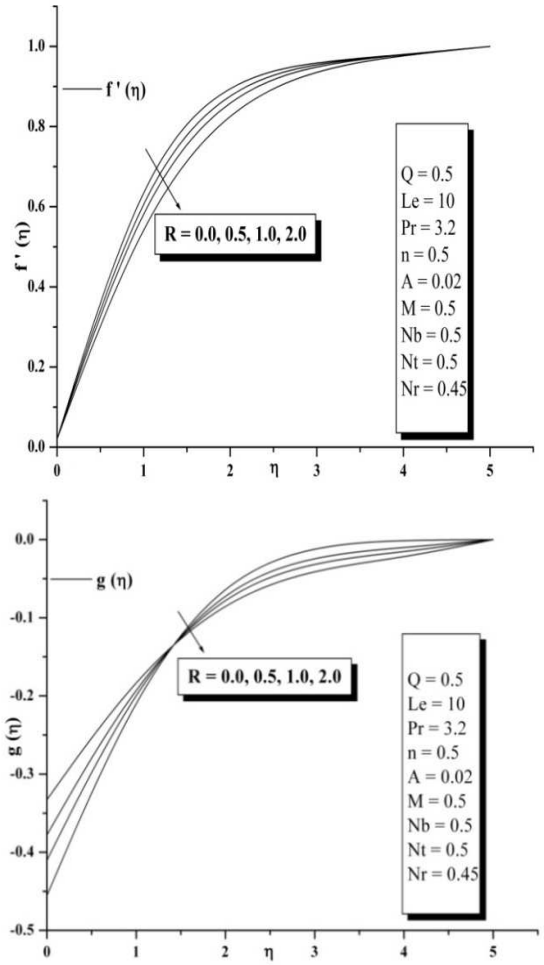
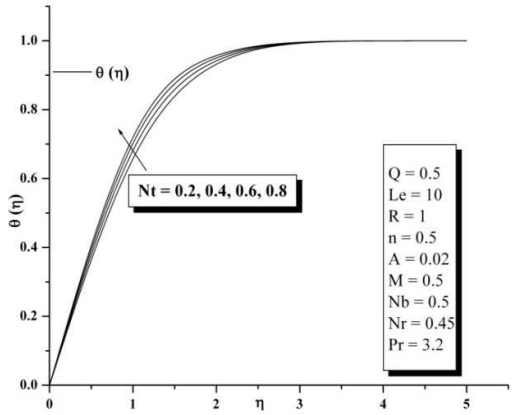


Figure 5: Effect of R on velocity and angular velocity profiles.



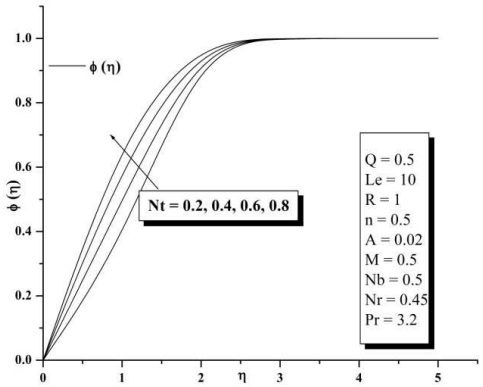


Figure 6: Effect of N_r on temperature and concentration profiles.

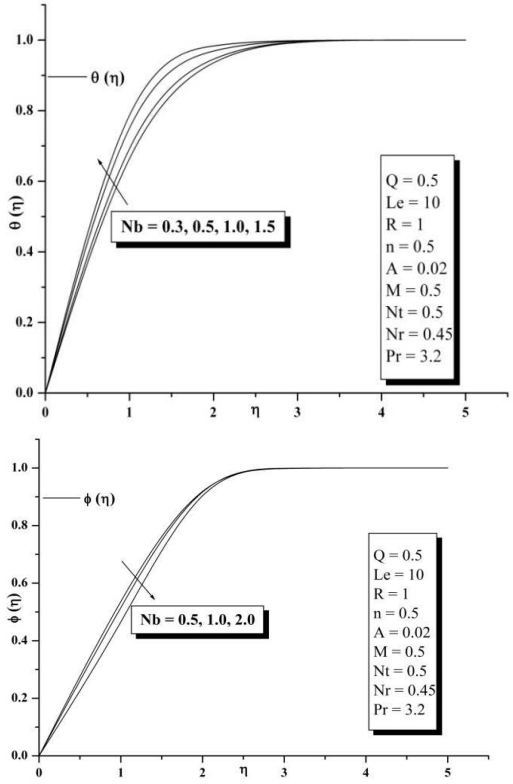


Figure 7: Effect of Nb on temperature and concentration profiles.

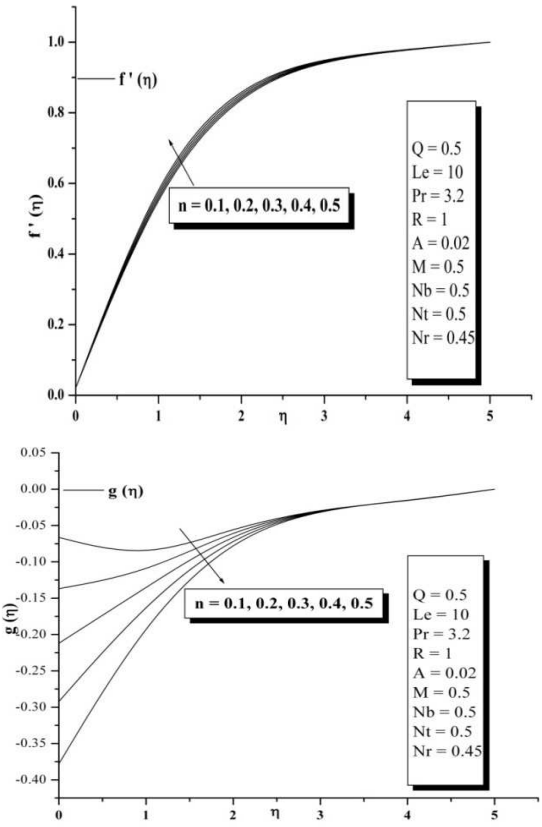
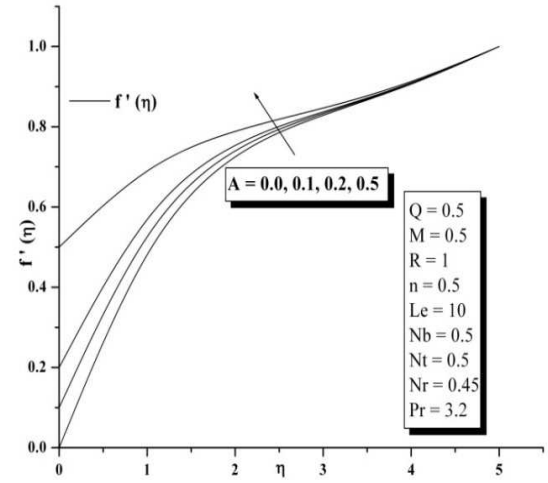


Figure 8: Effect of n on velocity and angular velocity profiles.



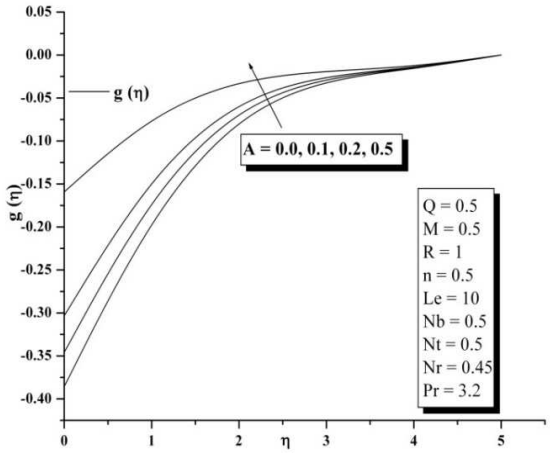


Figure 9: Effect of A on velocity and angular velocity profiles.

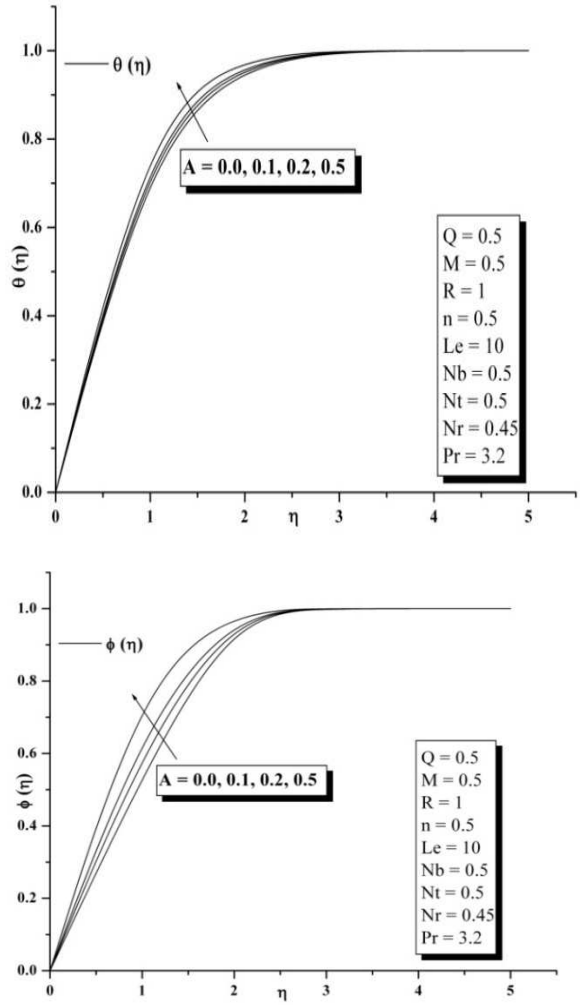


Figure 10:Effect of A on temperature and concentration profile.

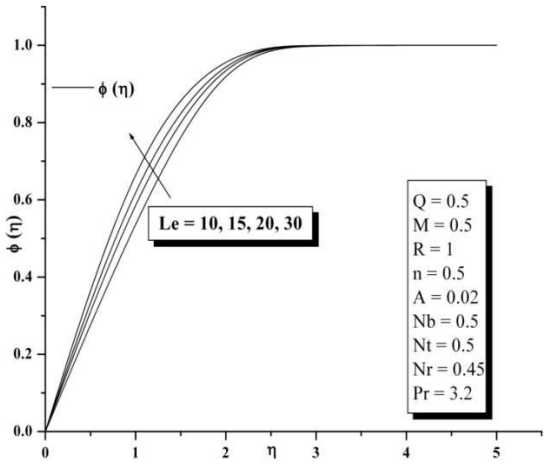


Figure 11: Effect of Le on concentration profiles

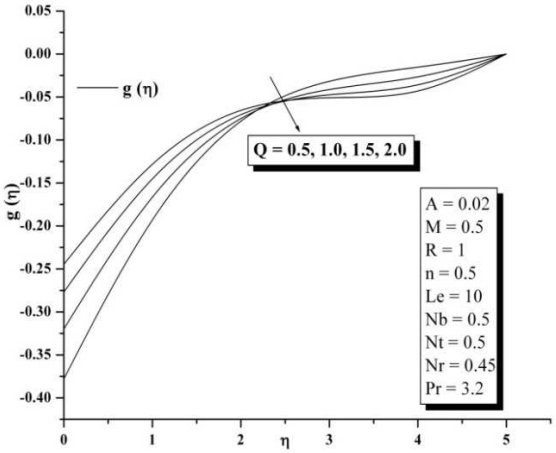


Figure 12: Effect of Q on angular velocity.

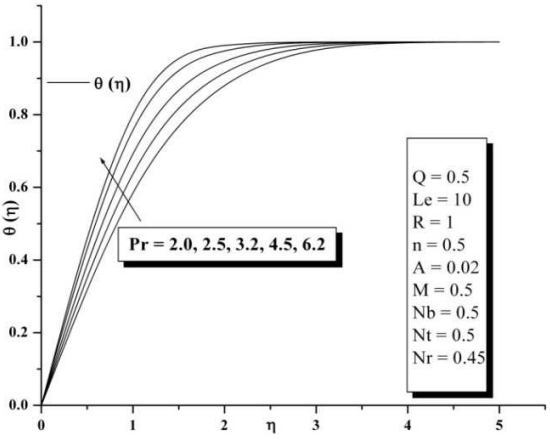
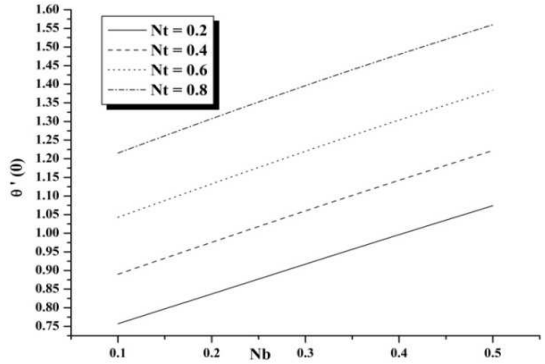


Figure13: Effect of Pr on temperature profiles.



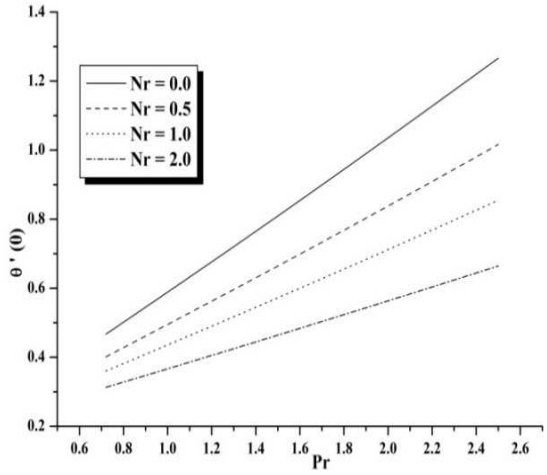
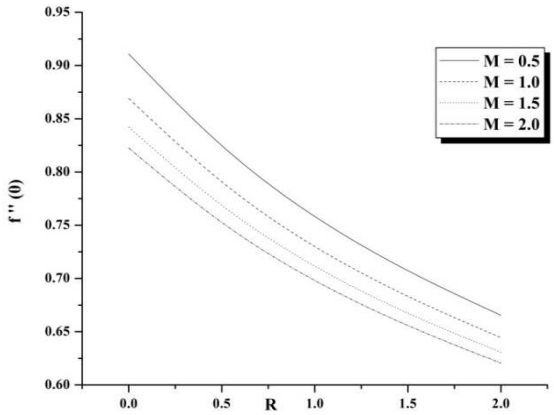


Figure 14 (a), (b): Variation of $\theta'(0)$ with Nb for different with Nt and variation with Pr for different values of Nr.



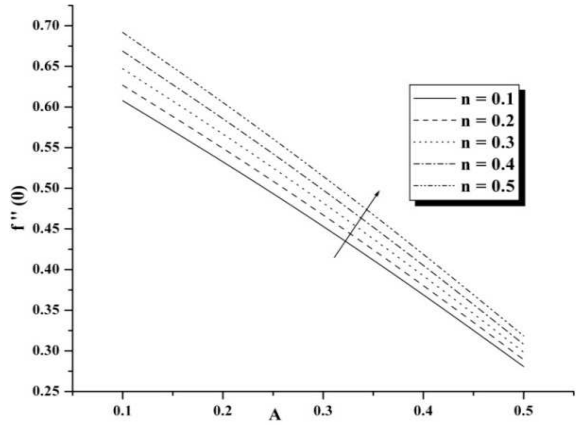


Figure 15(a), (b): Effect of skin friction coefficient with R for different values of M and with A for different values of n

Table 1: Comparison of present values of $f''(0)$ and $-\theta'(0)$ for different values of stretching parameter A and melting parameter M, when β, Ec, Q, R, $N = 0$ and $Pr = 1$.

parameters		Bachok et al [3]		Yacob et al [44]		Present study	
A	M	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.0	0.0	1.23525876		1.2325877	0.570465	1.23258768	0.570469995
	1.0	1.0370034		1.037003	0.361961	1.036997334	0.361913338
	2.0	0.9468506				0.946833343	0.273721264
	3.0	0.891381				0.89139121	0.223162584
0.5	0.0	0.7132949		0.713295	0.692064	0.713294966	0.692065815
	1.0	0.5990895		0.59909	0.438971	0.599088513	0.438971285
	2.0	0.5043333				0.5047017449	0.331927254
	3.0	0.5151721				0.515166211	0.270616779
1.0	0.0		0.7978846			0.713294966	0.797884176
	1.0		0.5060545				0.50605615
	2.0		0.3826383				0.38264128
	3.0		0.3119564				0.311960624
2.0	0.0	-1.8873066		-1.887307	0.979271	-1.88730663	0.979271368
	1.0	-1.5804839		-1.580483	0.621187	-1.58048373	0.62118726
	2.0	-1.4427473		-1.442747		-1.4427467	0.469711948
	3.0	-1.3592105				-1.35920926	0.382950899