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Forbidden Subgraphs for Planar Vertex Semi-Middle Graph

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Abstract

In this communication, we present characterizations of graphs whose vertex semi-middle graph $M_v(G)$ is planar, outerplanar and minimally nonouterplanar in terms of forbidden subgraphs. Further, we obtain $M_v(G)$ is not maximal planar.

Keywords: *Forbidden, Minimally nonouterplanar graph, Outerplanar graph, Planar graph, Vertex semi-middle graph.*

1. Introduction

A characterisation of graphs with a certain attribute by "forbidding" a certain family of subgraphs is normally of high interest due to its practical applicability. Greenwell and Hemminger [1] defined graphs with planar line graphs in terms of forbidden subgraphs. We will define a graph in this work as a nontrivial connected graph. We use the terminology of [2].

A graph is said to be planar if it can be drawn on the plane without any of its edges intersecting. A planar graph is outerplanar. IIf all of its vertices are on the exterior region, it can be embedded in the plane. The concept of a minimally nonouterplanar graph is first described in [3]. When considering a planar graph G, the inner vertex number i(G) is defined as the minimum possible number of vertices that do not belong to either the boundary of the exterior region or any of the boundaries of G in the plane. Assuming that i(G) = 0, then G is clearly planar. If i(G)=1, then a graph G is minimally nonouterplanar and G is k-minimally nonouterplanar (k≥2) if i(G)=k.

Consider a planar graph with R regions. The vertex semi-middle graph of a graph G, denoted by $M_v(G)$ is a graph whose vertex set is $V(G) \cup E(G) \cup R(G)$ and two vertices of $M_v(G)$ are adjacent if and only if they corresponds to two adjacent edges of G or one corresponds to a vertex and other to an edge incident with it or one corresponds to a vertex other to a region in which vertex lies on the region. This concept was introduced in [17].
The graph G and its vertex semi-middle graph $M_v(G)$ as shown in Fig.1. Many other graph valued functions in graph theory were studied, for example, in [4–16, 18-21].

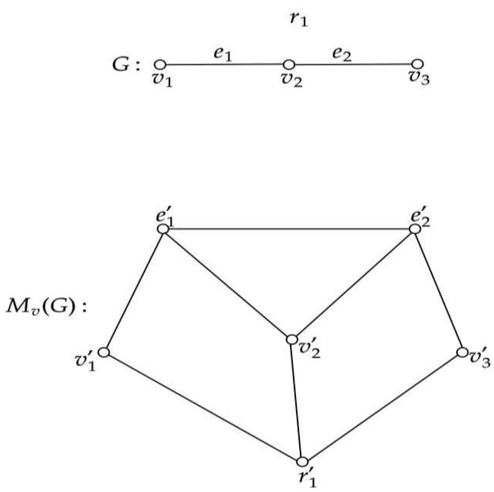

Figure 1: The graph G and its $M_v(G)$.

2. Preliminaries

Theorem 2.1. [17] For every planar graph G, $Cr[M_v(G)]=1$ if and only if G = C_3 or $K_{1,3}(P_{n_1}, P_{n_2}, P_{n_3})$, Where $n_1, n_2, n_3 \geq 0$.

Theorem 2.2. [17] For every planar graph G, $Cr[M_v(G)] = 2$ if and only if G = C_4 or $B_{2,2}$ or subdivision of any edge in $B_{2,2}$ or $C_3(P_{n_1})$, Where $n_1 \geq 1$.

Theorem 2.3. [17] For every graph G, $M_v(G)$ is planar if and only if G = P_n.

Theorem 2.4. [17] For every planar graph G, $M_v(G)$ is outerplanar if and only if G= P_2.

Theorem 2.5. [17] The $M_v(G)$ of a conneccted graph G is k-minimally nonouterplanar k ∈ I^+ if and only if G = P_{k+2}.

3. Main Results

Theorem 3.1. For every graph G, $M_v(G)$ is not maximal planar.

Proof. Since E(G) and R(G) are independent set of vertices of $M_v(G)$ and also it is possible to join at least two vertices of E(G) without loosing planarity. Therefore, $M_v(G)$ is not maximal planar.

Theorem 3.2. The vertex semi-middle graph $M_v(G)$ of a graph G is planar if and only if G has no subgraph homeomorphic to C_3 or $K_{1,3}$ or $B_{2,2}$.

Proof. Let $M_v(G)$ be planar. By Theorem 2.1, G has no subgraph homeomorphic to C_3. Suppose G is a $K_{1,3}$. By Theorem 2.1, G has no subgraph homeomorphic to $K_{1,3}$. Suppose G is $B_{2,2}$. By Theorem 2.2, G has no subgraph homeomorphic to $B_{2,2}$. Hence G has no subgraph homeomorphic to C_3 or $K_{1,3}$ or $B_{2,2}$.

On the other hand, assume G has no subgraph homeomorphic to C_3 or $K_{1,3}$ or $B_{2,2}$. Assume that G is a cycle of length greater than two, then G contains a subgraph homeomorphic to C_3, a contradiction. Suppose G is a path of length two adjoined to some vertices on degree of two, then G contains a subgraph homeomorphic to $K_{1,3}$. Suppose G is $K_{2,2}$ then G contains a subgraph homeomorphic to $B_{2,2}$. Then Clearly every block of G is a path by Theorem 2.3, $M_v(G)$ is planar.

Theorem 3.3. The $M_v(G)$ of a graph G is outerplanar if and only if G has no subgraph homeomorphic to P_3.

Proof. Assume that $M_v(G)$ is outerplanar. By Theorem 2.4, G has no subgraph homeomorphic to P_3.

On the other hand, assume G has no subgraph homeomorphic to P_3. Assume G is path of length greater than or equal to 4, then G contains a subgraph homeomorphic to P_3, a contradiction. Then G must be a outerplanar.

Theorem 3.4. The vertex semi-middle graph $M_v(G)$ of a graph G is minimally nonouterplanar if and only if G has no subgraph homeomorphic to P_2.

Proof. Suppose $M_v(G)$ is minimally nonouterplanar. By Theorem 2.5, G has no subgraph homeomorphic to P_2.

Conversely, suppose G has a subgraph homeomorphic to P_2. Then clearly G is P_3. By Theorem 2.5, $M_v(G)$ is minimally nonouterplanar.

Theorem 3.5. If G is a P_2, $M_v(G)$ is maximal outerplanar.

Proof. Suppose G is a P_2. By Theorem 2.4 $M_v(G)$ is C_4, which is a outerplanar.

Suppose G is a P_3. Then $M_v(G)$ is 1-minimally nonouterplanar. Hence for G is a P_2 then $M_v(G)$ is maximal outerplanar.

4. Conclusion

In this communication, we discuss the planar, outerplanar and minimally nonouterplanar of vertex semi-middle graph in terms of forbidden subgraphs. Also we discuss $M_v(G)$ is not maximal planar.

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